

Optics and Modern Physics

CHAPTER 5

Dual Nature of Radiation and Matter

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5

Dual Nature of Radiation and Matter

INTRODUCTION

While studying classical physics we learned that the laws that govern the characteristics of particles and waves are fundamentally different.

Louis de-Broglie propounded that under proper circumstances, particles can behave as waves while waves may behave as particles. Thus dual nature, i.e. particle and wave, was established and a fascinating branch of physics, called **quantum mechanics**, evolved. In quantum mechanics, some of the classically inspired notions of the distinction between particles and waves are discarded. Quantum mechanics (or wave mechanics) involves concepts that are often strange and counter to our intuition. Einstein was one of many who contributed to the development of a radical revision of our understanding of particle and wave character by developing quantum theory.

We shall now show that a particle or wave characteristic is not an inherent property but depends upon the type of experimental observation.

PHOTONS

The concept of photons and quanta comes from quantum mechanics and quantum theory. Quantum mechanics is a mathematical model that describes the behaviour of particles on an atomic and subatomic scale. It demonstrates that matter and energy are quantized, or come in small discrete bundles.

In 1900, Planck proposed that electromagnetic radiation (or light) is quantized and exists in elementary amounts (or quanta) that we now call photons. According to this proposal, the quantum of light wave of frequency f has the energy: $E = hf$.

Here h is Planck's constant and it has the value,

$$h = 6.63 \times 10^{-34} \text{ J s} = 4.14 \times 10^{-15} \text{ eV s}$$

We can say that the light energy from any source is always an integral multiple of a smaller energy value called quantum of light. Hence, energy:

$$Q = NE, \text{ where } N \text{ (number of photons)} = 1, 2, 3, \dots$$

Here energy is quantized. $E = hf$ is the quantum of energy, it is a packet of energy called photon.

$$\text{Also } E = hf = \frac{hc}{\lambda} \text{ (in joules)} \quad \dots(i)$$

So, the least energy a light of frequency f can have is hf . The light cannot have energy like $1.5hf$ or πhf .

The energy of a photon generally expressed in electron-volt (eV).

$$\text{In Eq. (i), } hc = 12400 \text{ (eV-}\text{\AA}\text{)} \Rightarrow E = \frac{12400}{\lambda(\text{in } \text{\AA})} \text{ eV}$$

PROPERTIES OF PHOTONS

A photon can be considered as a particle of electromagnetic radiation defined as a discrete bundle (or quantum) of electromagnetic (or light) energy. A photon describes the particle properties of an electromagnetic wave instead of the overall wave itself. In other words, we can picture an electromagnetic wave as being made up of individual particles called photons.

- A source of radiation emits energy in the form of photons and these photons travel in straight line with the speed of light. Photon is not a material particle, it is a packet of energy.
- Energy of a photon depends upon its frequency and it does not change with change in medium.
- With change in medium, the speed and wavelength of the photon change but frequency does not change.
- Photons are electrically neutral. They are not deflected by electric or magnetic fields.
- A photon can collide with material particles like electron. During these collisions, the total energy and total momentum remain constant.
- A photon does not exist at rest. Its rest mass is zero.
- Equivalent mass of photon: $E = mc^2 = hf$

$$\Rightarrow m = \frac{hf}{c^2} = \frac{h}{c\lambda}$$

- Photons have momentum: $p = mc = \frac{h}{c\lambda} c = \frac{h}{\lambda}$
- (Intensity of a light beam) $\propto$ (number of photons present in it).

ILLUSTRATION 5.1

What is the energy (in eV) of a photon of wavelength 12400 Å?

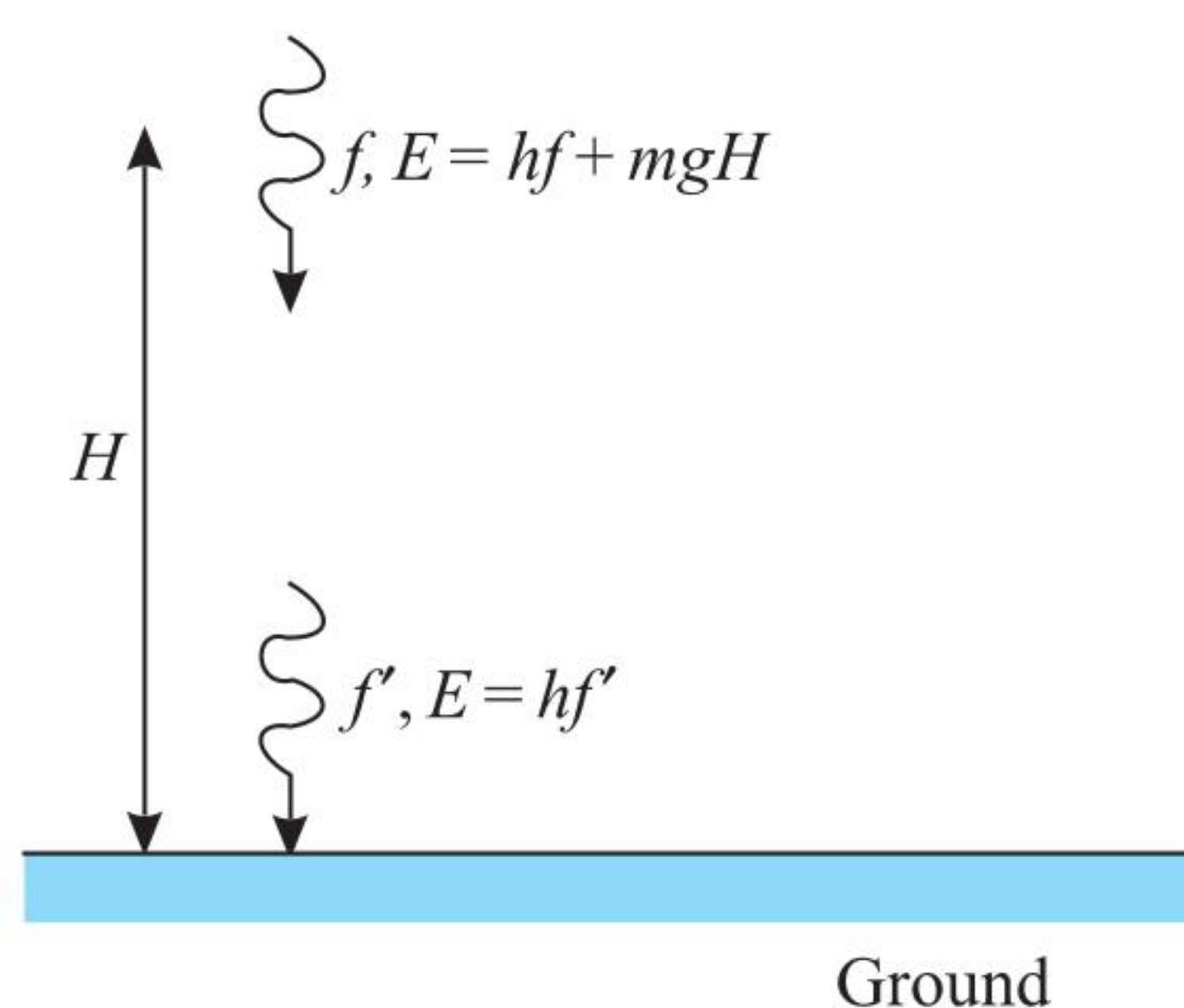
Sol. Using Plank's formula, we have $E = hf = \frac{hc}{\lambda}$

$$E = \frac{12400}{\lambda(\text{\AA})} \text{ eV} = \frac{12400}{12400} \text{ eV} = 1 \text{ eV}$$

ILLUSTRATION 5.2

If a photon frequency f falls on the surface of earth from a height H , then what will be its frequency on the surface of earth.

Sol. Change in frequency of the photon takes place.



Let f be the frequency of photon at height H .

Then the mass of photon, $m = \frac{hf}{c^2}$

Mass of the photon depends on its frequency, but we will consider the mass to be constant as difference in f' and f is very small.

Energy conservation gives Initial energy = final energy

$$\Rightarrow hf + mgH = hf'$$

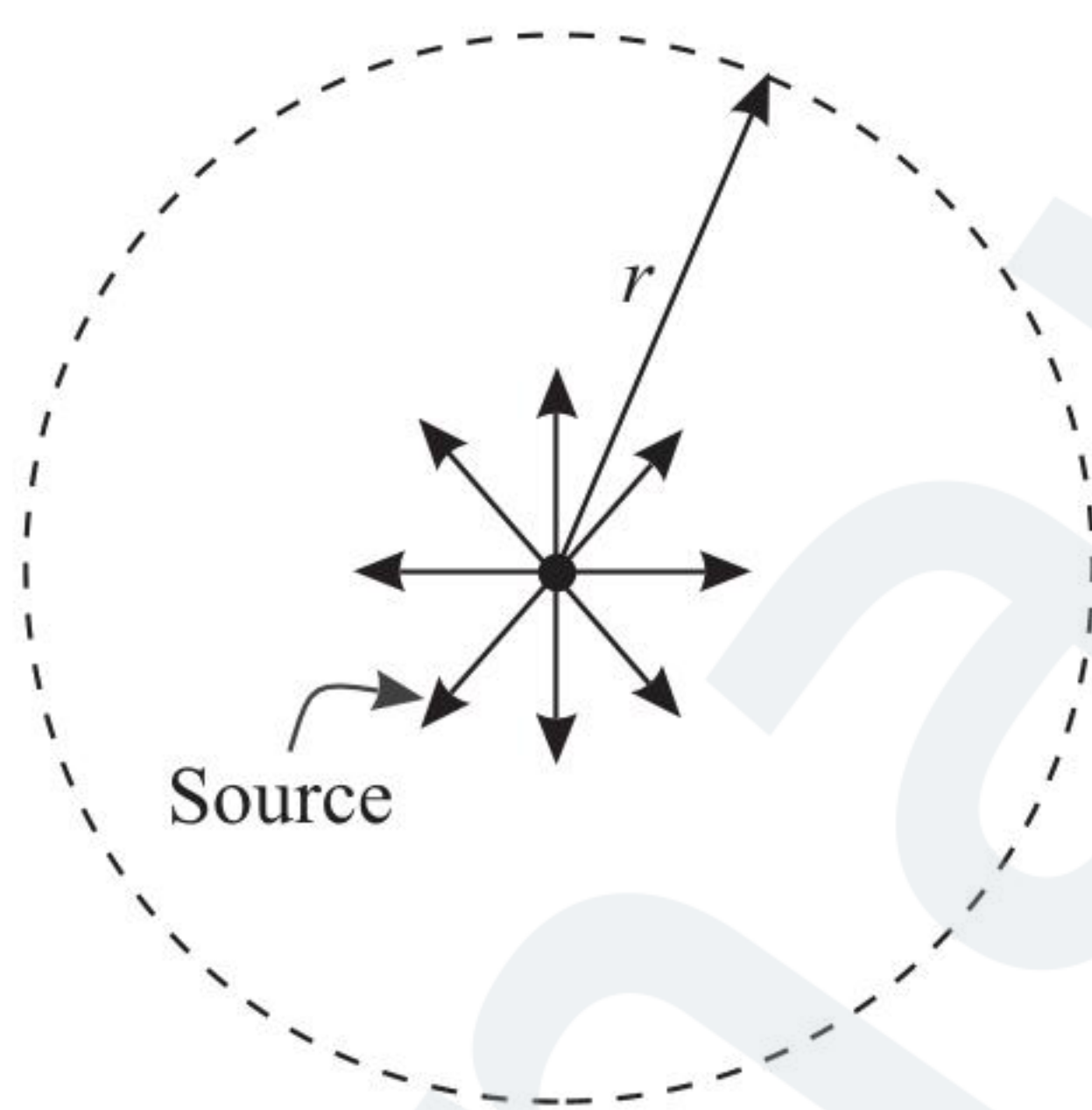
$$\Rightarrow hf + \left(\frac{hf}{c^2}\right)gH = hf'$$

$$\Rightarrow f' = f\left(1 + \frac{gH}{c^2}\right)$$

PHOTON COUNTS EMITTED BY A SOURCE PER SECOND

Consider a point source of power P watt as the source of radiation energy (see figure). If the wavelength of radiation emitted by the source is λ , then energy of each photon emitted by the source can be given as

$$E = hf = \frac{hc}{\lambda} \quad \dots(i)$$



The power of the source is P watt, we can say that the source is emitting radiation energy P joule per second in the form of photons. If it is 100% efficient, then the number of photons emitted per second by the source can be given as

$$n = \frac{\text{Power of the source}}{\text{Energy of photon}} = \frac{P}{E} = \frac{P\lambda}{hc} \quad \dots(ii)$$

These photons are assumed to be emitted uniformly in all directions. Here, we can consider that all the light energy emitted by the source is uniformly distributed in the spherical region with center at the source.

ILLUSTRATION 5.3

Calculate the number of photons emitted in 10 h by a 60 W sodium lamp ($\lambda = 6200 \text{ \AA}$).

Sol. The energy of a photon (in eV) can be given by

$$E = \frac{12400}{\lambda(\text{\AA})} \text{ eV} = \frac{12400}{6200} = 2 \text{ eV}$$

Hence the energy of a photon in joules,

$$E = 2 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-19} \text{ J}$$

Therefore, the number of photons emitted by sodium lamp in 10 h

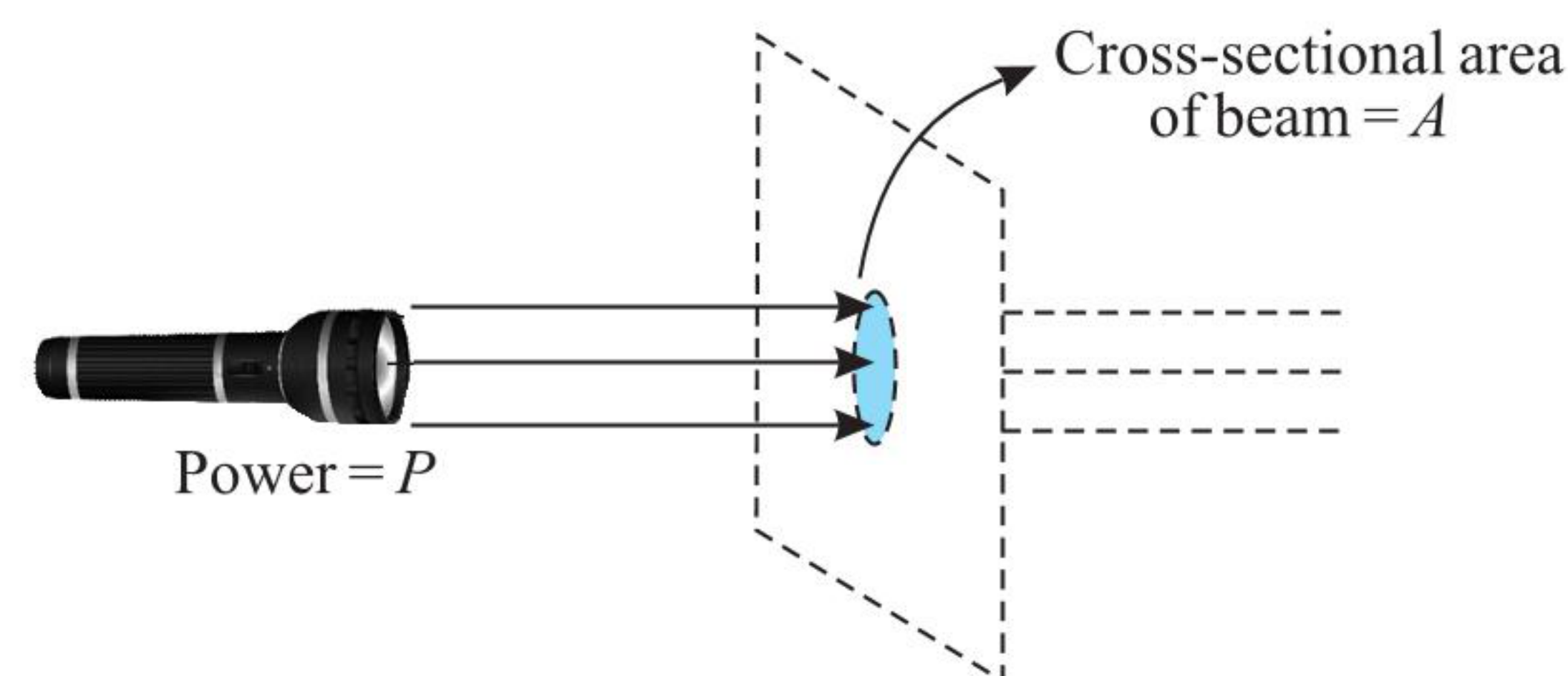
$$N = \frac{\text{Total energy released in 10 h}}{\text{Energy of a photon}}$$

$$\Rightarrow N = \frac{60 \times 10 \times 3600}{3.2 \times 10^{-19}} = 6.75 \times 10^{24}$$

INTENSITY OF RADIATION

The radiation energy crossing per unit area per unit time normal to the direction of propagation is called the **intensity of a wave (radiation)**.

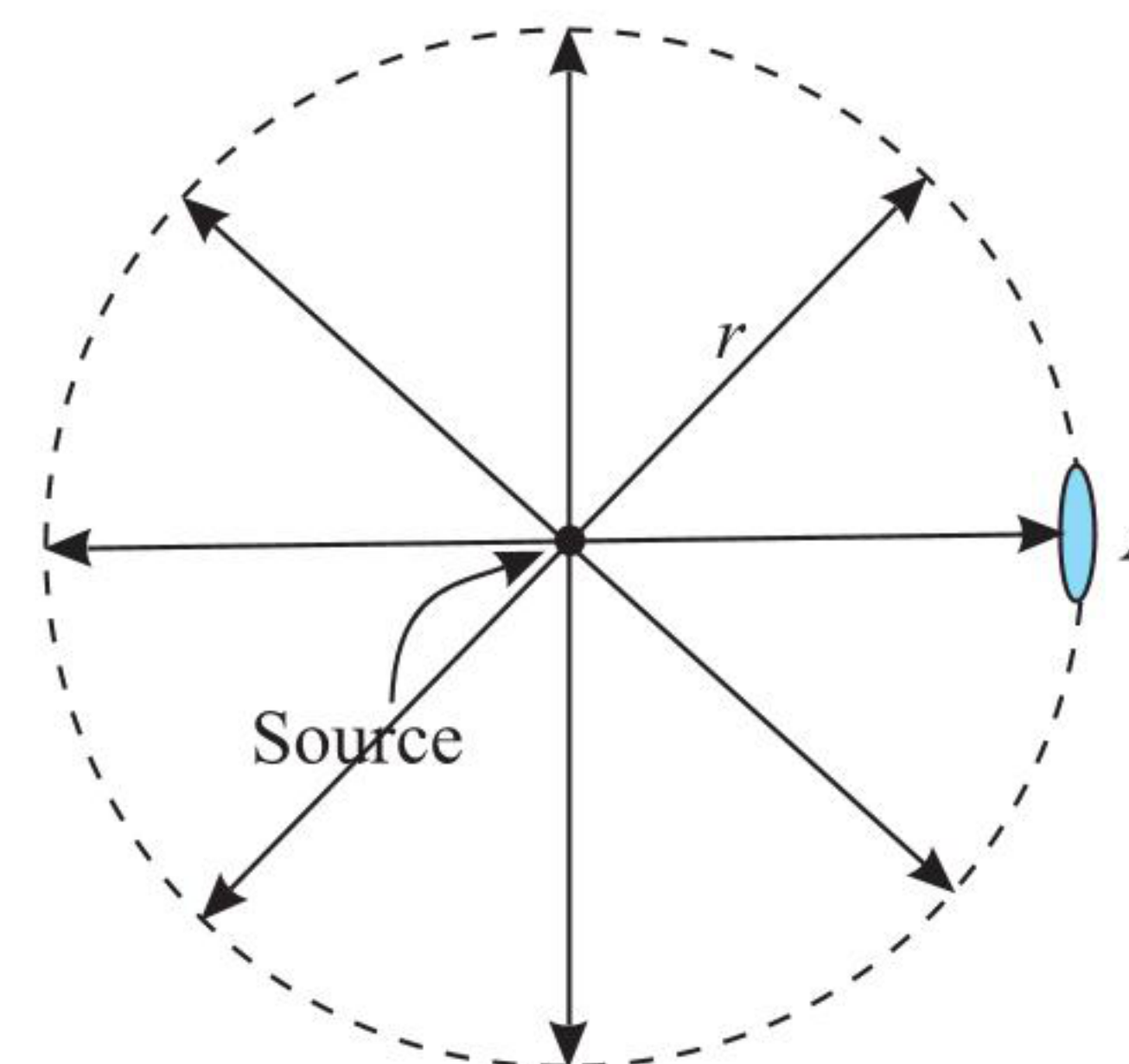
Consider a source of light emits a uniform cylindrical light beam of cross-sectional area A . If the source emits a total power P in the beam as shown in figure.



then the intensity of light beam is $I = \frac{P}{A} \text{ W m}^{-2}$

If cross-sectional area of the beam remain constant throughout, the beam intensity at every point will remain constant.

Now let us find the intensity of light (radiation) due to a point isotropic source as shown in figure.



If the power of source is P watt emitting radiation in all directions uniformly. If we want to calculate intensity of light at a point, at a distance r from the source, then it can be given as

$$I = \frac{P}{4\pi r^2} \text{ W m}^{-2}$$

Here, we can assume that the whole power P is incident on the normal area of a hypothetical sphere of radius r passing through points with center at the source as shown.

Note: The radiation power (P') crossing through a small area A at a distance r , $P' = IA = \left(\frac{P}{4\pi r^2}\right)A$

PHOTON FLUX

It is defined as the number of photons incident on a normal surface per second per unit area. Consider a light beam of intensity $I \text{ W m}^{-2}$ having wavelength λ incident on a surface. Then, number of photons per second per unit area in the beam can be given as

$$\text{Photon flux, } \phi = \frac{\text{Intensity}}{\text{Energy of a photon}} = \frac{I}{hc/\lambda} = \frac{I\lambda}{hc} \quad \dots(i)$$

If we consider a point source of power P watt which emits light in all directions, then it produces photons per second at a rate

$$n = \frac{\text{Power of a source}}{\text{Energy of a photon}} = \frac{P}{(hc/\lambda)} = \frac{P\lambda}{hc} \quad \dots(ii)$$

All these photons are distributed in the three-dimensional spherical space around the source. If we find the photon flux at a distance r from the point source, it can be given as

$$\phi = \frac{\text{Number of photons per second}}{\text{Surface area of sphere of radius } r} = \frac{n}{4\pi r^2} \quad \dots(iii)$$

ILLUSTRATION 5.4

A bulb lamp emits light of mean wavelength of $4500 \text{ \AA}$. The lamp is rated at 150 W and 8% of the energy appears as emitted light. How many photons are emitted by the lamp per second?

Sol. The energy of photon $= \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{4500 \times 10^{-10}}$

According to the given problem, power of the lamp is 150 W . As 8% of the light energy is utilized for emission of light, hence the utilized energy for emission per second is given by

$$\text{Useful power} = 150 \times \frac{8}{100} = 12 \text{ W} = 12 \text{ J s}^{-1}$$

Therefore, number of photons emitted per second by the lamp

$$\frac{\text{Useful power}}{\text{Energy of photon}} = \frac{12 \times 4500 \times 10^{-10}}{(6.63 \times 10^{-34})(3 \times 10^8)} = 27.17 \times 10^{18}$$

ILLUSTRATION 5.5

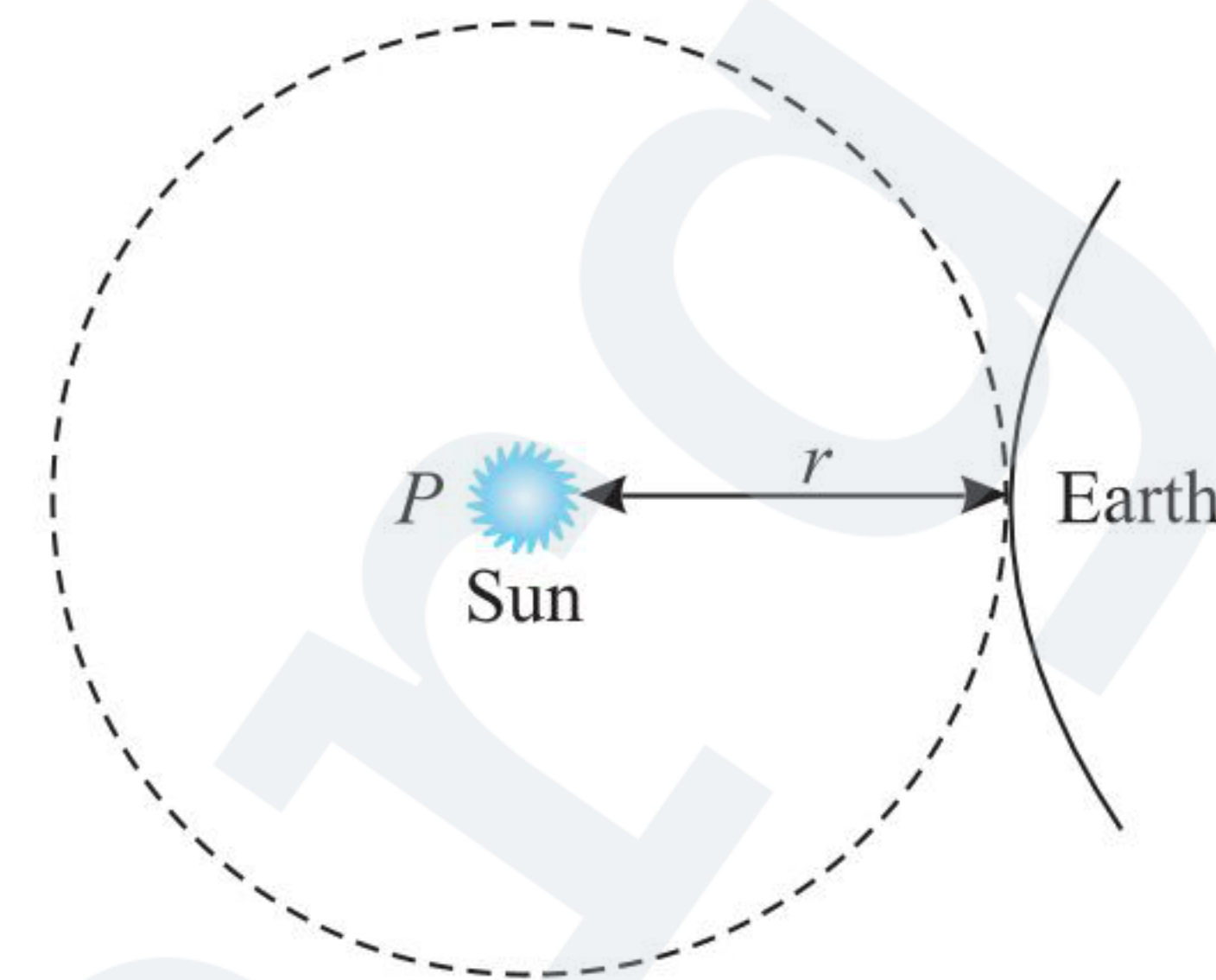
The power of light emitted by the sun is $3.9 \times 10^{26} \text{ W}$. Assuming the mean wavelength of sunlight to be $6000 \text{ \AA}$, calculate the photon flux arriving at earth perpendicular to light radiations. The average radius of earth's orbit around sun is $1.49 \times 10^{11} \text{ m}$.

Sol. Power of light emitted by the sun, $P = 3.9 \times 10^{26} \text{ W}$

Earth sun mean distance $r = 1.49 \times 10^{11} \text{ m}$

Mean wavelength of sun light, $\lambda = 6000 \text{ \AA} = 6 \times 10^{-7} \text{ m}$

Intensity of light on earth, $I = \frac{P}{4\pi r^2}$



The energy of a photon, $E = \frac{hc}{\lambda}$

Hence photon-flux, $\phi = \frac{I}{hc/\lambda} = \frac{P\lambda}{4\pi r^2 hc}$

$$\Rightarrow \phi = \frac{(3.9 \times 10^{26}) \times (6 \times 10^{-7})}{4\pi (1.49)^2 \times 10^{22} \times 6.63 \times 10^{-34} \times 3 \times 10^8}$$

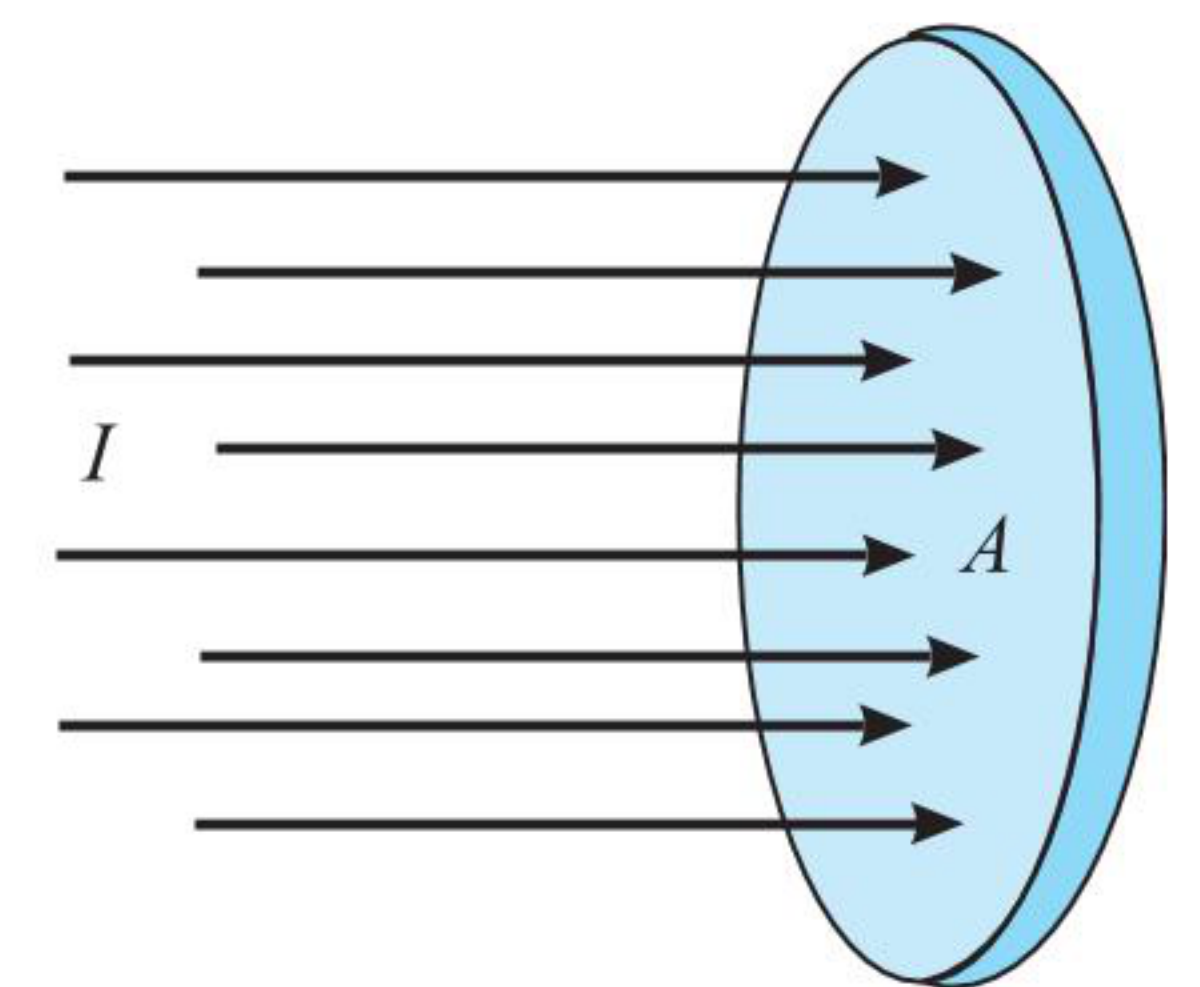
$$\text{or } \phi = 4.22 \times 10^{21} \text{ photon/s m}^2$$

FORCE EXERTED ON A SURFACE DUE TO RADIATION

When photons fall on a surface, they exert a force and pressure on the surface. The force (pressure) experienced by surface exposed to radiation is known as radiation force (pressure). This force can be calculated using photon theory of radiation.

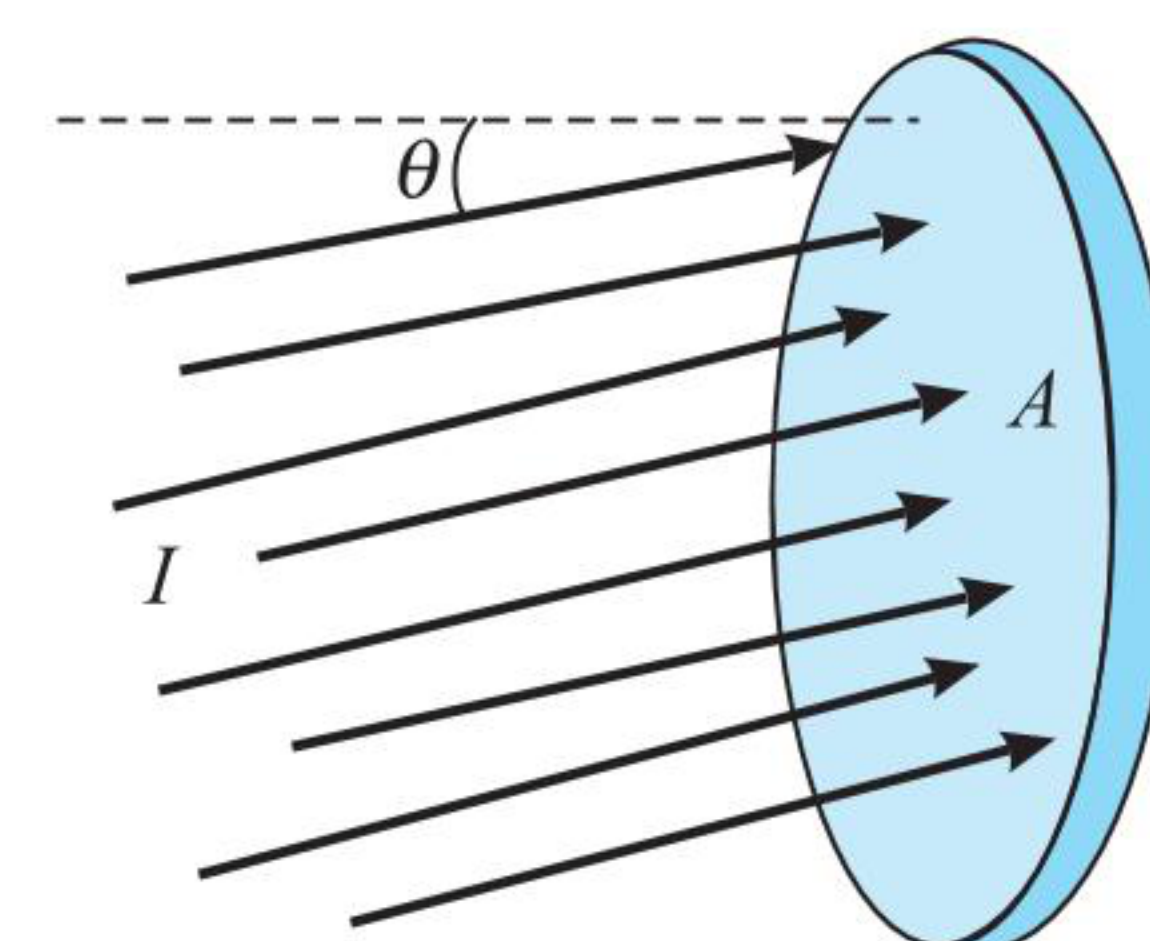
Let us consider a surface of area A exposed to radiation of intensity I as shown in figure. The radiation is falling normally on the surface.

Assume absorption and reflection coefficients of surface are a and r , respectively. The absorption and reflection coefficients means the fraction of the photons being absorbed and reflected by the incident surface respectively. Assume no transmission. Energy received by the surface per second, $P = IA$. It is also the power received by the surface. Number of photons received by surface per second,



$$n = \frac{P}{\text{Energy of one photon}} = \frac{P}{hc/\lambda} = \frac{IA\lambda}{hc}$$

If radiation incident on the surface at certain angle, then power incident on the surface, $P = IA \cos \theta$

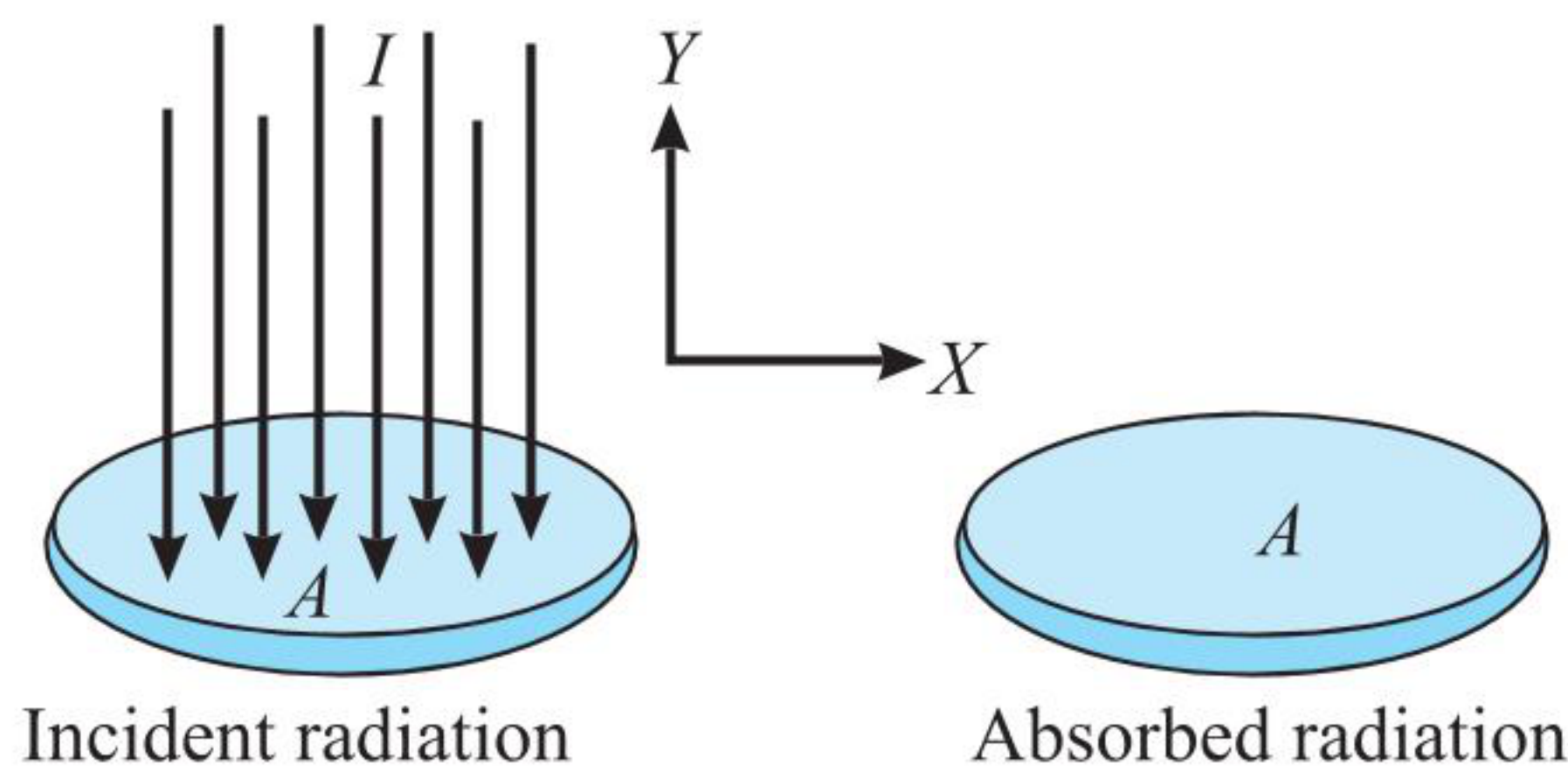


The average force imparted by radiation can be expressed

$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

Here $\Delta \vec{p}$ is the change in momentum of the photons incident on the surface.

Case I. The radiation falling on the surface normally and absorbed by the surface: For this, absorption coefficient $a = 1$, reflection coefficient $r = 0$.



The momentum of a photon $p = \frac{h}{\lambda}$

Initial momentum of one photon $\vec{p}_{\text{initial}} = \left(\frac{h}{\lambda}\right)(-\hat{j})$

(i.e. in downward direction)

Final momentum of the photon $\vec{p}_{\text{final}} = 0$

Change in momentum of one photon,

$$\Delta \vec{p} = \vec{p}_{\text{final}} - \vec{p}_{\text{initial}} = 0 - \left(\left(\frac{h}{\lambda}\right)(-\hat{j})\right) = \frac{h}{\lambda}(\hat{j})$$

(i.e. in upward direction)

Energy incident per unit time (or power), $P = IA$

Number of photons incident per unit time, $n = \frac{IA}{hf} = \frac{IA\lambda}{hc}$

Therefore, the total change in momentum of photons per unit time, it is also force acting on photons

$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t} = n \frac{h}{\lambda}(\hat{j}) = \frac{IA\lambda}{hc} \times \frac{h}{\lambda}(\hat{j}) = \frac{IA}{c}(\hat{j})$$

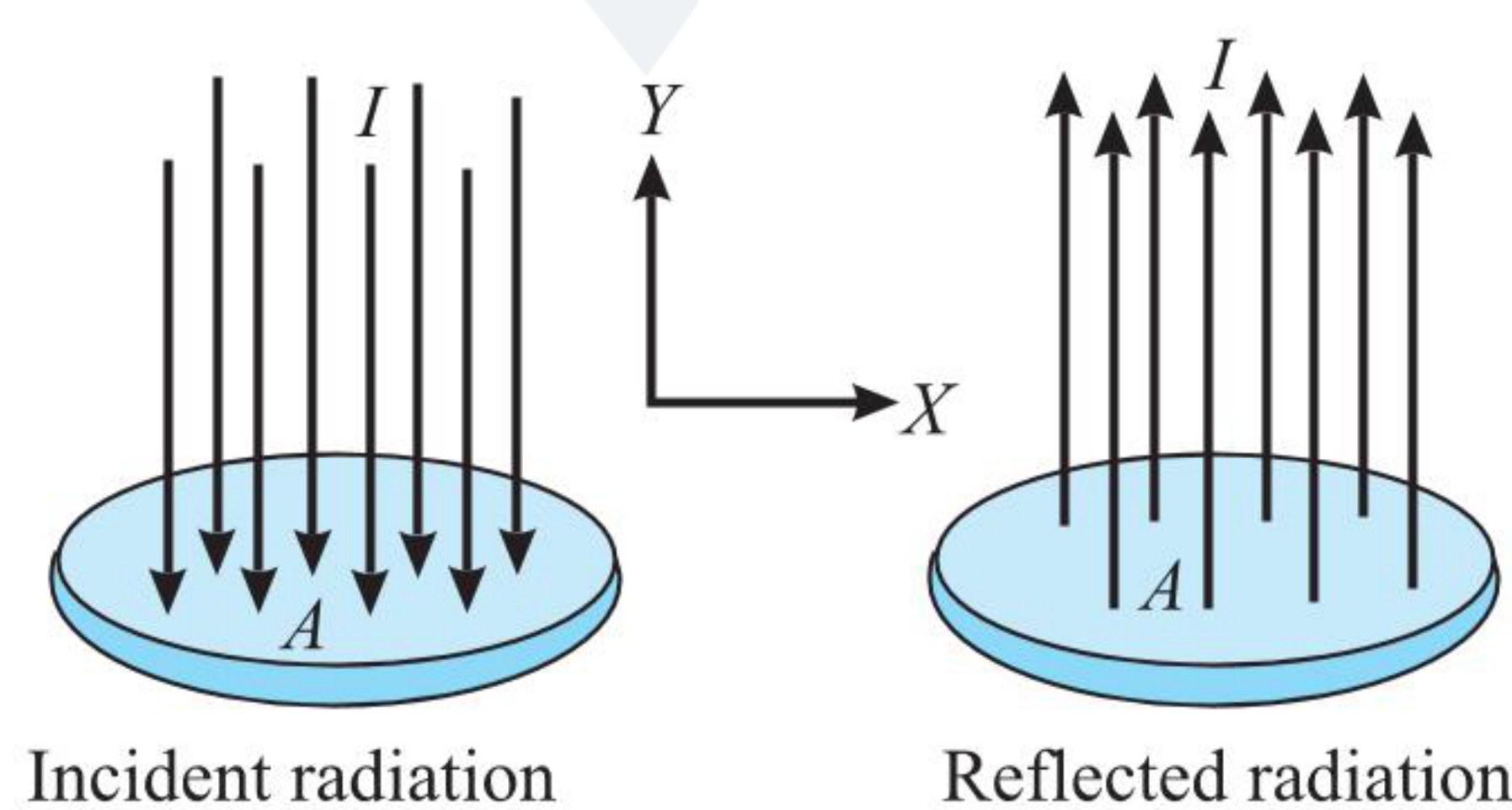
(i.e. in upward direction),

Therefore, force on plate due to photons,

$$F = \frac{IA}{c} \quad (\text{downward})$$

Therefore, radiation pressure $= \frac{F}{A} = \frac{IA}{cA} = \frac{I}{c}$

Case II. The radiation falling on the surface normally and reflected by the surface: For this, reflection coefficient $r = 1$, absorption coefficient $a = 0$.



Initial momentum of one photon, $\vec{p}_{\text{initial}} = \left(\frac{h}{\lambda}\right)(-\hat{j})$
(i.e. in downward direction)

Final momentum of the photon, $\vec{p}_{\text{final}} = \left(\frac{h}{\lambda}\right)(\hat{j})$

Change in momentum of one photon,

$$\Delta \vec{p} = \vec{p}_{\text{final}} - \vec{p}_{\text{initial}} = \left(\frac{h}{\lambda}\right)(\hat{j}) - \left(\frac{h}{\lambda}\right)(-\hat{j}) = \frac{2h}{\lambda}(\hat{j})$$

(i.e. in upward direction)

Energy incident per unit time (or power) $p = IA$

Number of photons incident per unit time, $n = \frac{IA}{hf} = \frac{IA\lambda}{hc}$

Therefore, the total change in momentum of photons per unit time, it is also force acting on photons

$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t} = n \frac{2h}{\lambda}(\hat{j}) = \frac{IA\lambda}{hc} \times \frac{2h}{\lambda}(\hat{j}) = \frac{2IA}{c}(\hat{j})$$

(i.e. in upward direction)

Therefore, force on plate due to photons, $F = \frac{2IA}{c}$ (downward)

Hence radiation pressure, $P = \frac{F}{A} = \frac{2IA}{cA} = \frac{2I}{c}$

Case III. When some part of radiation is absorbed and the remaining part is reflected: For this, reflection coefficient r , absorption coefficient a .

Force on plate due to reflected photons, $F_r = r \left(\frac{2IA}{c}\right)$
(downward direction)

Force on plate due to absorbed photons, $F_a = a \left(\frac{IA}{c}\right)$
(downward direction)

Total force on plate, $F = F_r + F_a = r \frac{2IA}{c} + a \frac{IA}{c}$ (downward)

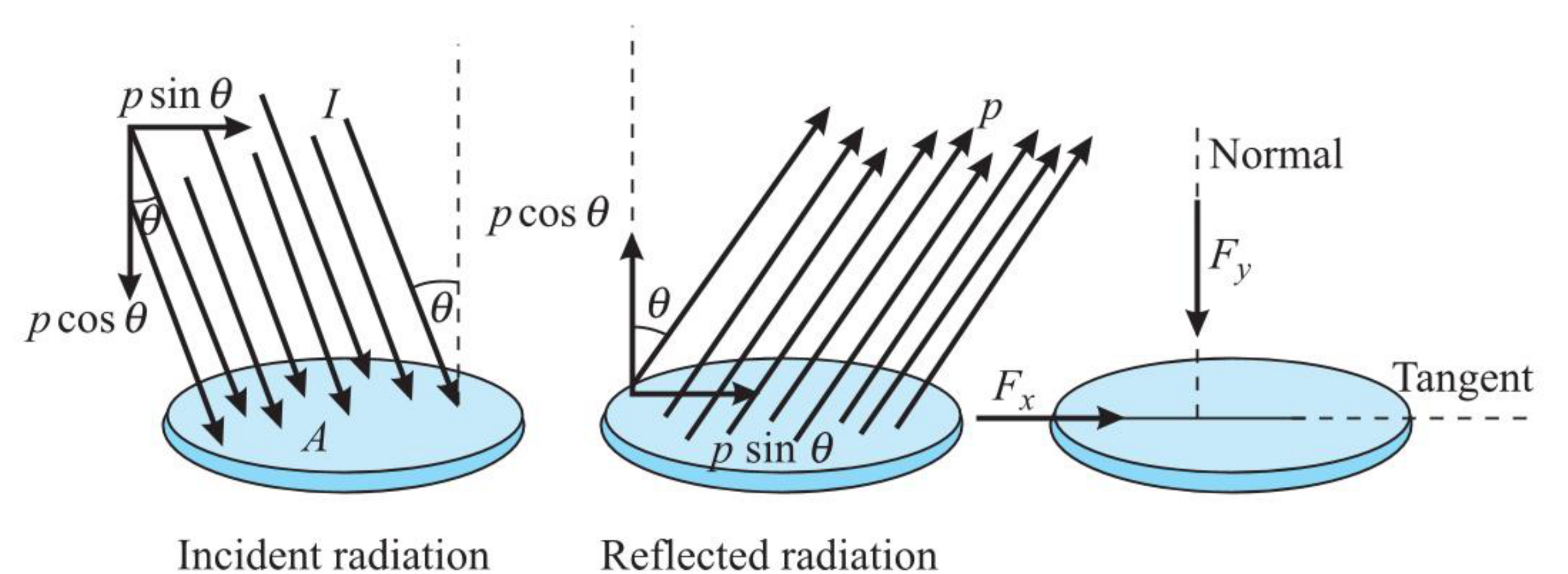
But, $a + r = 1$, so $a = 1 - r$

Hence, $F = r \frac{2IA}{c} + (1 - r) \frac{IA}{c} = (1 + r) \frac{IA}{c}$ (downward)

Hence radiation pressure, $P = \frac{F}{A} = \frac{I}{c}(1 + r)$

Case IV. The radiation falling on the surface at certain angle and reflected by the surface: In this case the momentum of the photons will remain unchanged in horizontal direction (in x-direction)

Hence, $F_x = 0$



Now considering in vertical direction:

The momentum of a photon, $p = \frac{h}{\lambda}$

Initial momentum of one photon, $(\vec{p}_{\text{initial}})_y = \left(\frac{h}{\lambda} \cos \theta\right)(-\hat{j})$
(i.e. in downward direction)

Final momentum of the photon, $(\vec{p}_{\text{final}})_y = \left(\frac{h}{\lambda} \cos \theta\right)(\hat{j})$

Change in momentum of one photon in vertical direction,

$$(\Delta \vec{p})_y = (\vec{p}_{\text{final}})_y - (\vec{p}_{\text{initial}})_y$$

$$(\Delta \vec{p})_y = \left(\frac{h}{\lambda} \cos \theta\right)(\hat{j}) - \left[\left(\frac{h}{\lambda} \cos \theta\right)(-\hat{j})\right] = \frac{2h}{\lambda} \cos \theta (\hat{j})$$

(i.e. in upward direction)

Energy incident per unit time (or power), $E = IA \cos \theta$

Number of photons incident per unit time,

$$n = \frac{IA \cos \theta}{hf} = \frac{IA \cos \theta \cdot \lambda}{hc}$$

Therefore, the total change in momentum of photons per unit time, i.e., force acting on photons,

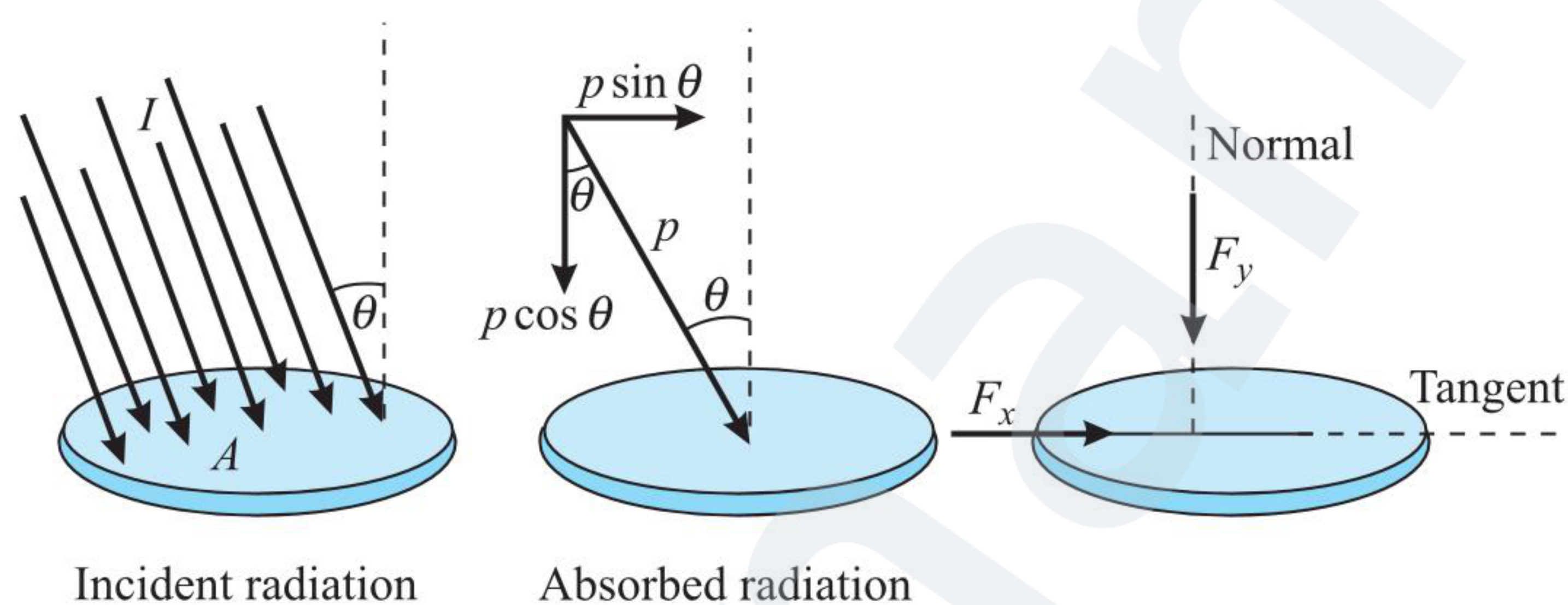
$$\vec{F}_y = \frac{\Delta \vec{p}_y}{\Delta t} = n \frac{2h}{\lambda} \cos \theta (\hat{j})$$

$$\Rightarrow \vec{F}_y = \frac{IA \cos \theta \cdot \lambda}{hc} \times \frac{2h}{\lambda} \cos \theta (\hat{j}) = \frac{2IA}{c} \cos^2 \theta (\hat{j})$$

(i.e. in upward direction),

$$\text{Hence radiation pressure, } P = \frac{F}{A} = \frac{2IA}{cA} \cos^2 \theta = \frac{2I}{c} \cos^2 \theta$$

Case V: The radiation falling on the surface at certain angle and absorbed by the surface: In this case the momentum of the photons will change in horizontal direction (x -direction) and vertical direction (y -direction).



Momentum of a photon, $p = \frac{h}{\lambda}$

Considering in horizontal direction.

Initial momentum of one photon, $(\vec{p}_{\text{initial}})_x = \left(\frac{h}{\lambda} \sin \theta\right)(\hat{i})$
(i.e. in rightward direction)

Final momentum of the photon, $(\vec{p}_{\text{final}})_x = 0$

Change in momentum of one photon in horizontal direction,

$$(\Delta \vec{p})_x = (\vec{p}_{\text{final}})_x - (\vec{p}_{\text{initial}})_x$$

$$(\Delta \vec{p})_x = 0 - \left[\left(\frac{h}{\lambda} \sin \theta\right)(\hat{i})\right] = -\frac{h}{\lambda} \sin \theta (\hat{i})$$

(i.e. in leftward direction)

Therefore, the total change in momentum of photons per unit time, i.e. force acting on photons,

$$\vec{F}_x = \frac{\Delta \vec{p}_x}{\Delta t} = n \left(\frac{h}{\lambda} \sin \theta\right)(-\hat{i})$$

$$\Rightarrow \vec{F}_x = \frac{IA \cos \theta \cdot \lambda}{hc} \times \left(\frac{h}{\lambda} \sin \theta\right)(-\hat{i}) = \frac{IA}{c} \sin \theta \cos \theta (-\hat{i})$$

(i.e. in leftward direction)

Hence the horizontal component of the radiation force on the surface is $\frac{IA}{c} \sin \theta \cdot \cos \theta$

Considering in vertical direction:

Initial momentum of one photon, $(\vec{p}_{\text{initial}})_y = \left(\frac{h}{\lambda} \cos \theta\right)(-\hat{j})$
(i.e. in downward direction)

Final momentum of the photon, $(\vec{p}_{\text{final}})_y = 0$

Change in momentum of one photon in vertical direction,

$$(\Delta \vec{p})_y = (\vec{p}_{\text{final}})_y - (\vec{p}_{\text{initial}})_y$$

$$(\Delta \vec{p})_y = 0 - \left[\left(\frac{h}{\lambda} \cos \theta\right)(-\hat{j})\right] = \frac{h}{\lambda} \cos \theta (\hat{j})$$

(i.e. in upward direction)

Energy incident per unit time (or power), $E = IA \cos \theta$

Number of photons incident per unit time,

$$n = \frac{IA \cos \theta}{hf} = \frac{IA \cos \theta \cdot \lambda}{hc}$$

Therefore, the total change in momentum of photons per unit time, i.e. force acting on photons,

$$\vec{F}_y = \frac{\Delta \vec{p}_y}{\Delta t} = n \frac{h}{\lambda} \cos \theta (\hat{j})$$

$$\Rightarrow \vec{F}_y = \frac{IA \cos \theta \cdot \lambda}{hc} \times \frac{h}{\lambda} \cos \theta (\hat{j}) = \frac{IA}{c} \cos^2 \theta (\hat{j})$$

(i.e. in upward direction)

Hence the vertical component of the radiation force on the surface will be $\frac{I}{c} \cos^2 \theta$ in downward direction.

$$\text{Hence radiation pressure, } P = \frac{F}{A} = \frac{IA}{cA} \cos^2 \theta = \frac{I}{c} \cos^2 \theta$$

ILLUSTRATION 5.6

- How many photons of a radiation of wavelength $\lambda = 5 \times 10^{-7} \text{ m}$ must fall per second on a blackened plate in order to produce a force of $6.62 \times 10^{-5} \text{ N}$?
- At what rate will the temperature of plate rise if its mass is 19.86 kg and specific heat is equal to $2500 \text{ J kg}^{-1} \text{ K}^{-1}$?

Sol.

- If n is the number of photons falling per second on the plate, then the total momentum per second of the incident photons is $p = n \times \frac{h}{\lambda}$.

Since the plate is blackened, all photons are absorbed by it.

$$\therefore \frac{\Delta P}{\Delta t} = n \frac{h}{\lambda}$$

$$\text{Since, } F = \frac{\Delta P}{\Delta t} = n \frac{h}{\lambda} \Rightarrow n = \frac{F\lambda}{h}$$

$$\text{or } n = \frac{6.62 \times 10^{-5} \times 5 \times 10^{-7}}{6.62 \times 10^{-34}} = 5 \times 10^{22}$$

$$(b) \text{ Energy of each photon} = \frac{hc}{\lambda}$$

Since n photons fall on the plate per second, the total energy absorbed by the plates in one second is

$$E = n \times \frac{hc}{\lambda} = 1986 \text{ J s}^{-1}$$

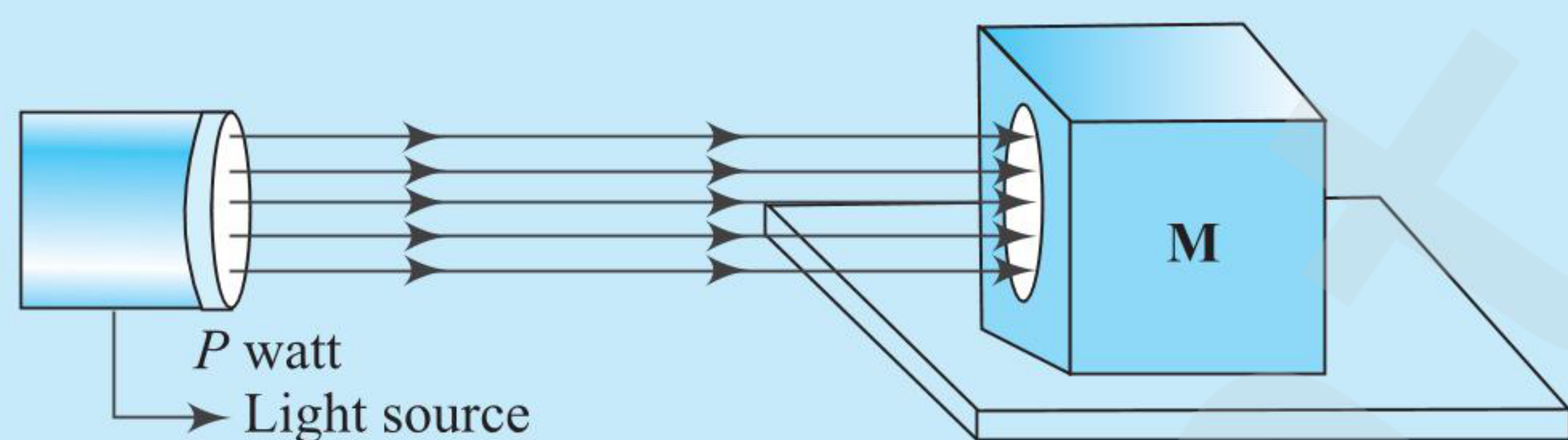
$$\text{i.e., } \frac{dQ}{dt} = 1986 \text{ J s}^{-1}$$

$$mc \frac{dT}{dt} = 1986$$

$$\Rightarrow \frac{dT}{dt} = 1986 / (19.86 \times 2500) = 4 \times 10^{-2} \text{ } ^\circ\text{C s}^{-1}$$

ILLUSTRATION 5.7

A source of light of power P is shown in figure. Find the force on the block placed in the path of the light rays. The surface of body on which the light beam is incident is having a reflection coefficient $a_r = 0.7$ and absorption coefficient $a_a = 0.3$.

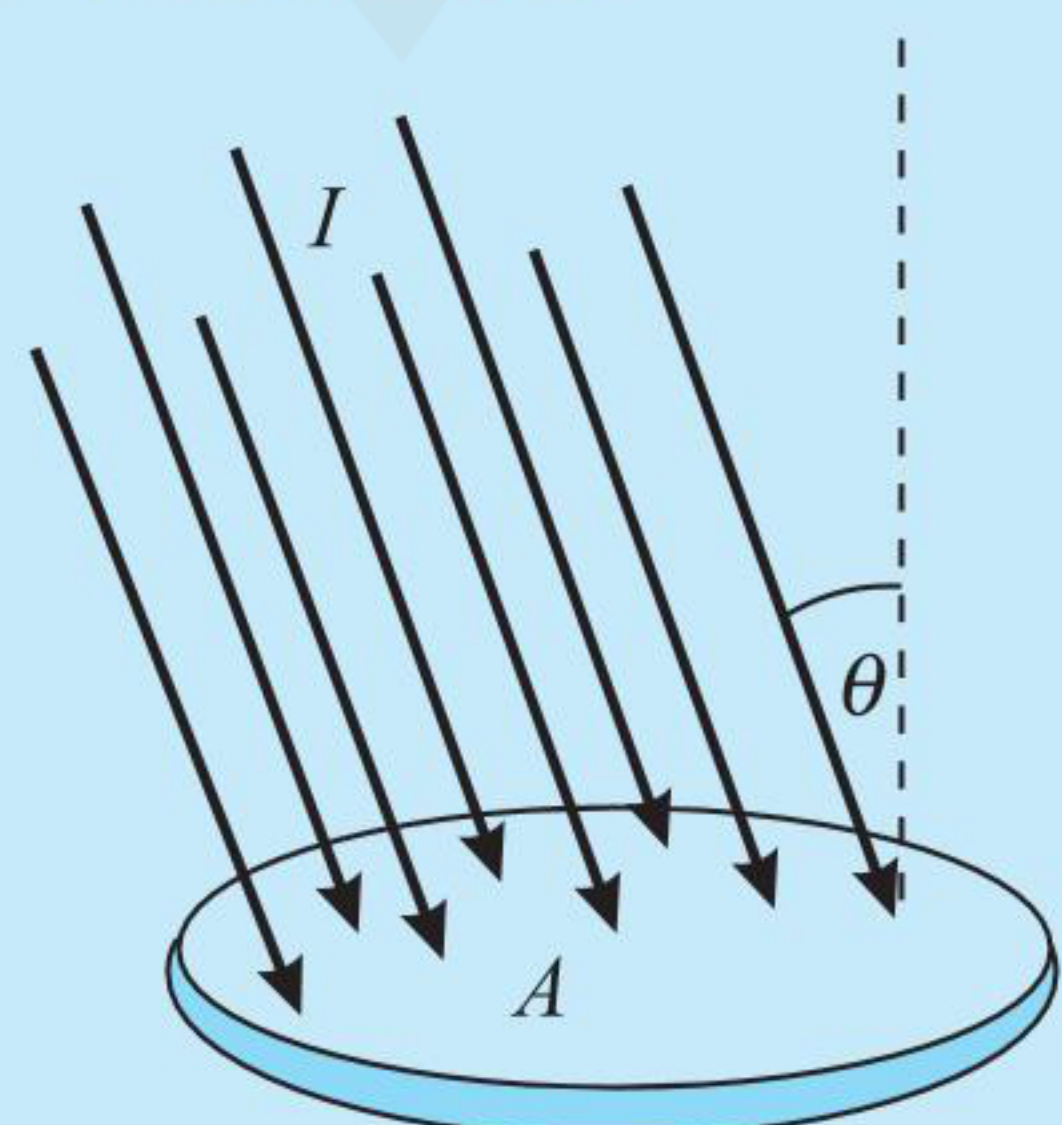


Sol. In this case, 70% of the incident photons are reflected back and 30% are absorbed by the body. Thus, the photon which is absorbed will impart a momentum h/λ to the body and the photon which is reflected will impart the change in momentum $2h/\lambda$ to the body. Thus, the net force acting on body can be given as

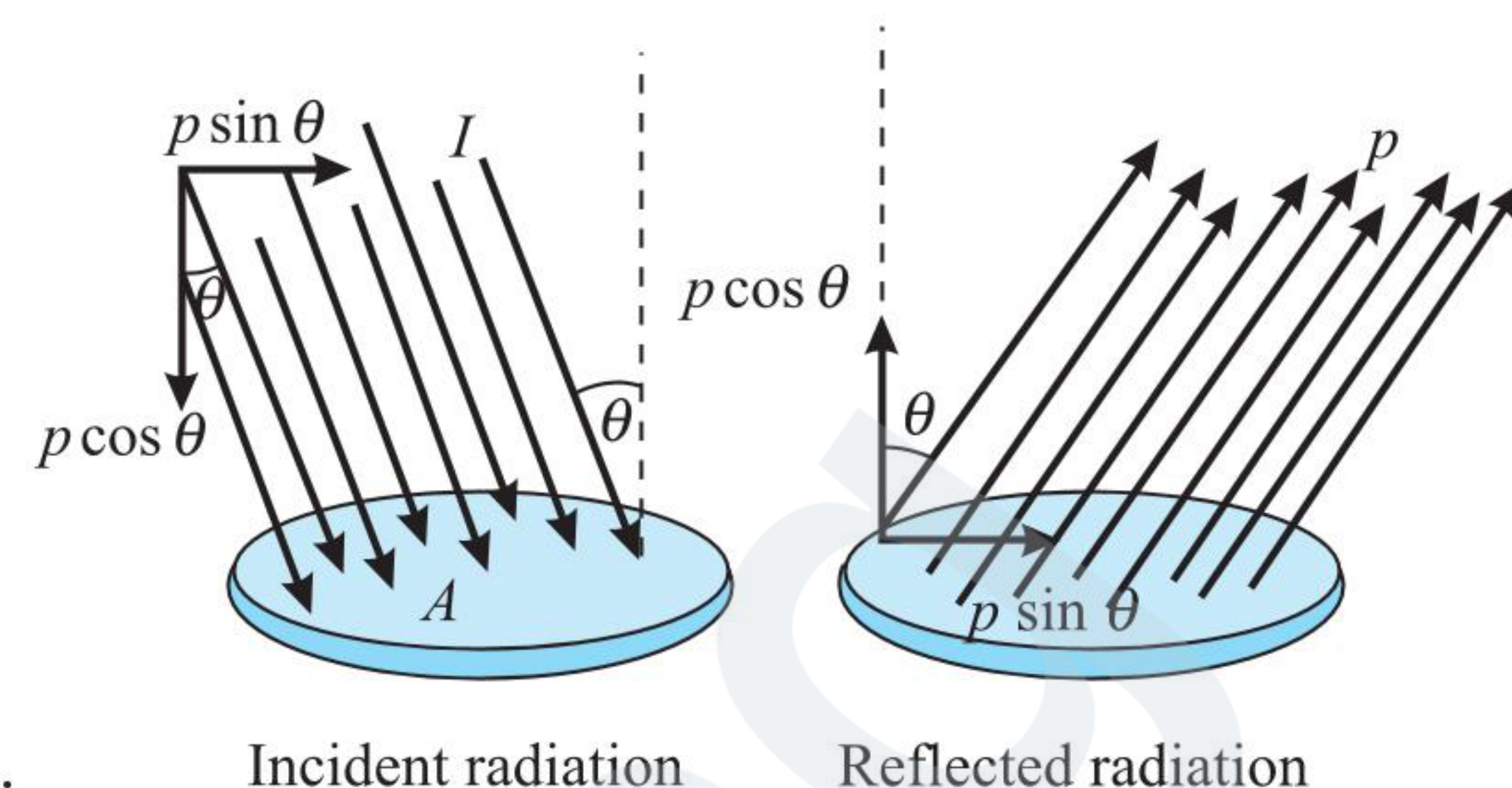
$$F = \frac{0.7P\lambda}{hc} \times \frac{2h}{\lambda} + \frac{0.3P\lambda}{hc} \times \frac{h}{\lambda} = \frac{1.7P}{c}$$

ILLUSTRATION 5.8

A plane light wave of intensity I falls on a plane mirror surface with reflection coefficient ρ . The angle of incidence is θ . In terms of corpuscular theory, find the magnitude of the normal pressure exerted on that surface.



Sol. The normal component of the radiation force will be responsible for radiation pressure.



The momentum corresponding to incident photons normal to the surface,

$$\left(\frac{dp}{dt} \right)_{\text{incident}} = \frac{I}{c} dA \cos^2 \theta$$

Since reflection coefficient is ρ , so the momentum of the reflected photons per second normal to surface,

$$\left(\frac{dp}{dt} \right)_{\text{reflected}} = -\frac{I}{c} dA \rho \cos^2 \theta$$

Hence, rate of change of momentum of the photons,

$$\left(\frac{dp}{dt} \right)_{\text{photons}} = -\frac{I}{c} dA (\rho + 1) \cos^2 \theta$$

From Newton's third law, $\left(\frac{dP}{dt} \right)_{\text{surface}} = -\left(\frac{dp}{dt} \right)_{\text{photons}}$

Hence, pressure exerted on surface, $P = \frac{dF}{dA} = \frac{I}{c} (\rho + 1) \cos^2 \theta$

ILLUSTRATION 5.9

Figure shows a small, plane strip of mass m suspended from a fixed support through a string. A continuous beam of monochromatic light is incident horizontally on the strip and is completely absorbed. The energy falling on the strip per unit time is W . Find the deflection of the string from the vertical, if the strip stays in equilibrium.

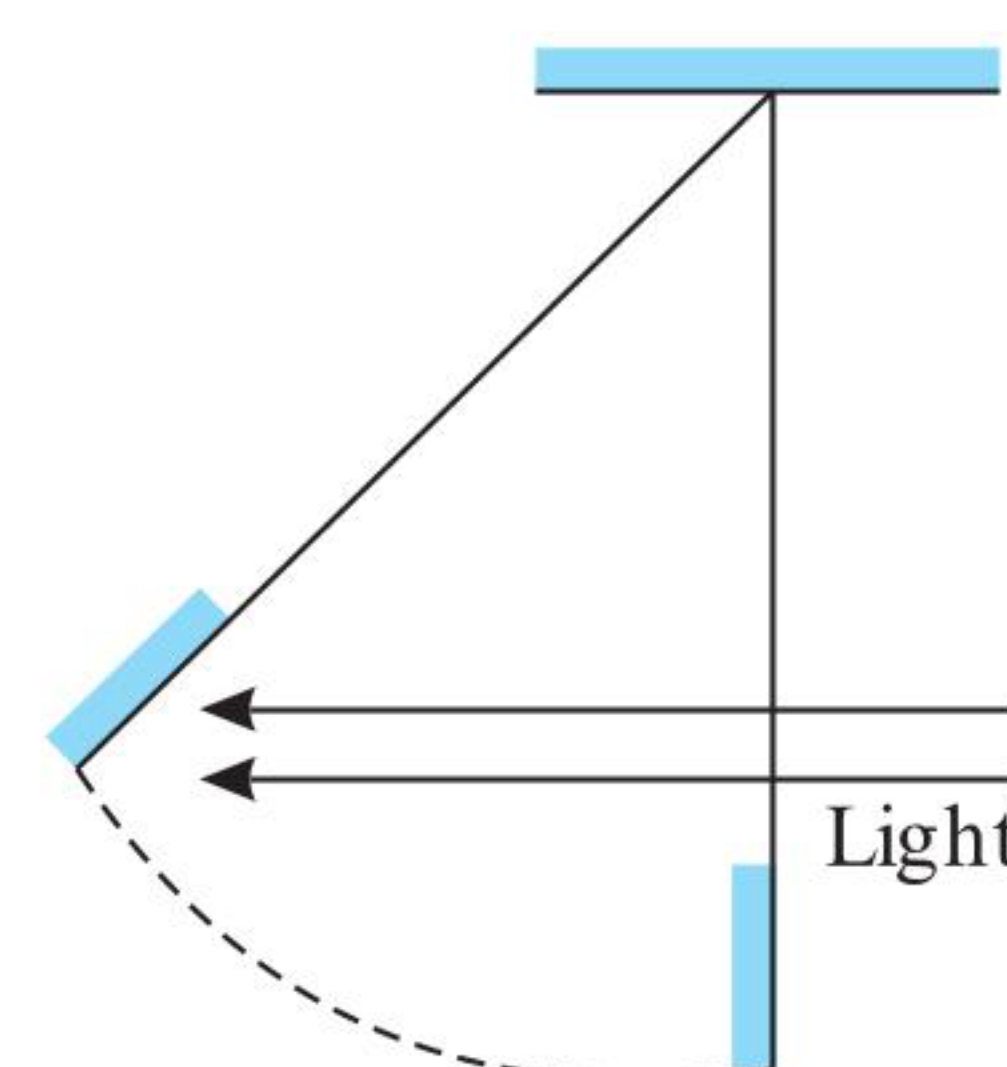
Sol. The radiation incident on the strip apply force on it, because of this force the strip is deflected from its original position. Force exerted by the photons of light = (number of incident photons per second) $\times$ (change in momentum of each photon)

Let n be the number of incident photons per second.

The momentum of photon incident on the strip per second,

$$P_{\text{incident}} = \frac{W}{c}$$

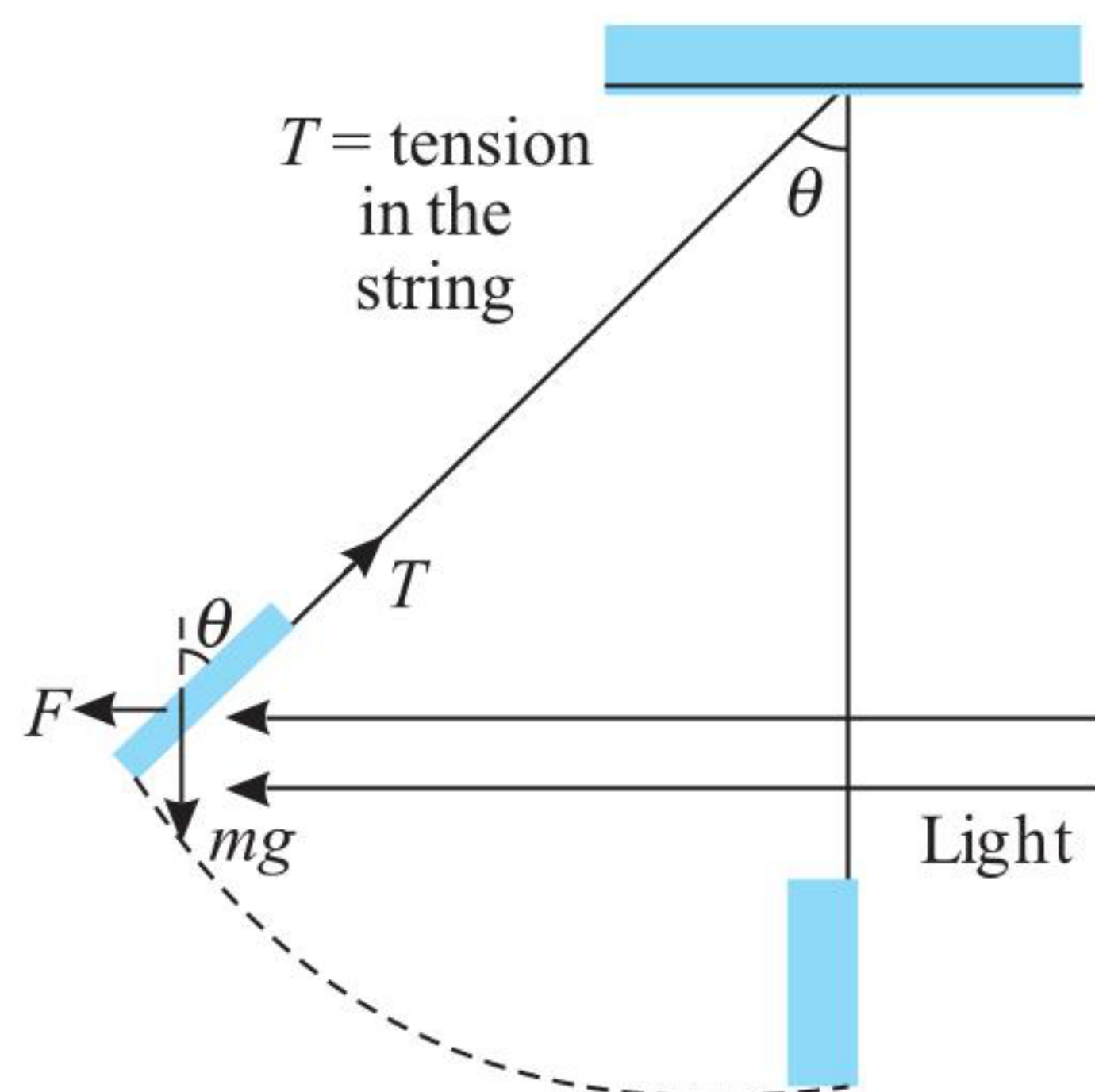
As the photon is completely absorbed, hence its final momentum should be zero.



The change in momentum of each photon per second, $\left| \frac{\Delta p}{\Delta t} \right| = \frac{W}{c}$

Force acting on the strip due to radiation, $F = \frac{W}{c}$

If the string makes an angle θ with the vertical when the strip is at equilibrium, then for the equilibrium of the strip,



$T \sin \theta = F$, in horizontal direction, and $T \cos \theta = mg$, in vertical direction.

Dividing the above equations, we get

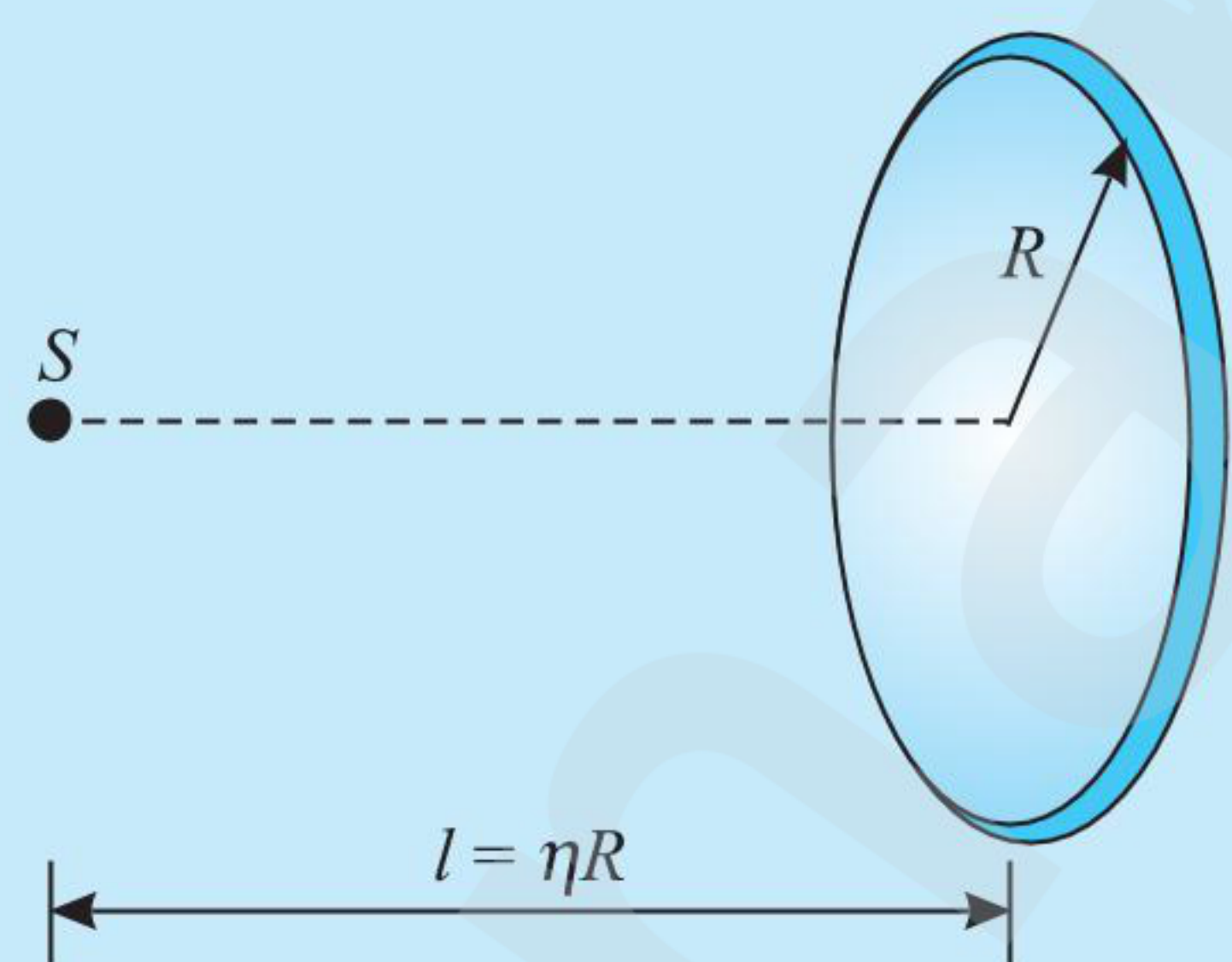
$$\tan \theta = \frac{F}{mg} = \frac{W/c}{mg} = \frac{W}{cmg}$$

$$\therefore \tan \theta = \frac{W}{cmg}$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{W}{cmg} \right)$$

ILLUSTRATION 5.10

An isotropic point source having radiation power P is located on the axis of an ideal mirror plate. The distance between the source and the plate exceeds the radius of the plate η -fold. Find the force that light exerts on the plate.



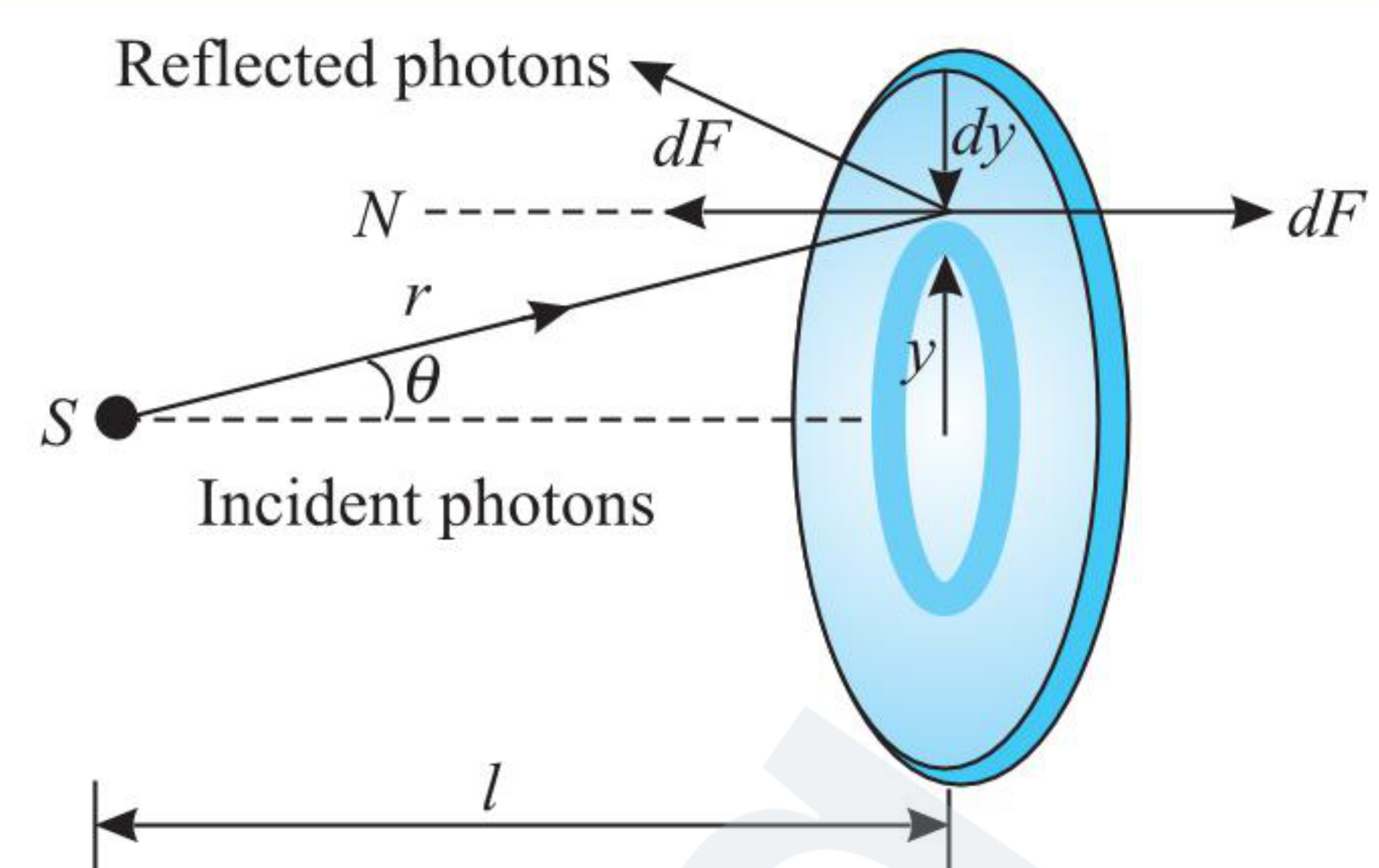
Sol. Intensity of radiation at distance of r from source,

$$I = \frac{P}{4\pi r^2}; \text{ here } P \text{ is the power of the source} \quad \dots(i)$$

If radius of the mirror is R and distance of the point source from the mirror is l , then according to the example, $\frac{l}{R} = \eta$.

Momentum per second of photons striking the mirror in normal direction,

$$\left(\frac{dp}{dt} \right)_{n, \text{incident}} = \frac{I}{c} dA \cos^2 \theta$$



Momentum per second of reflected photons,

$$\left(\frac{dp}{dt} \right)_{n, \text{reflected}} = -\frac{I}{c} dA \rho \cos^2 \theta$$

where ρ is reflection coefficient.

Hence rate of change of momentum of photons,

$$\left(\frac{dp}{dt} \right)_{n, \text{photons}} = -\frac{I}{c} dA(\rho + 1) \cos^2 \theta$$

From Newton's third law, rate of change of momentum of mirror,

$$\left(\frac{dp}{dt} \right)_{n, \text{mirror}} = \frac{I}{c} dA(\rho + 1) \cos^2 \theta$$

Since mirror plate is perfect reflector, force on its surface is

$$dF = \frac{2I}{c} dA \cos^2 \theta = \frac{2I}{c} 2\pi y dy \cos^2 \theta$$

where dA is the area of differential ring element,

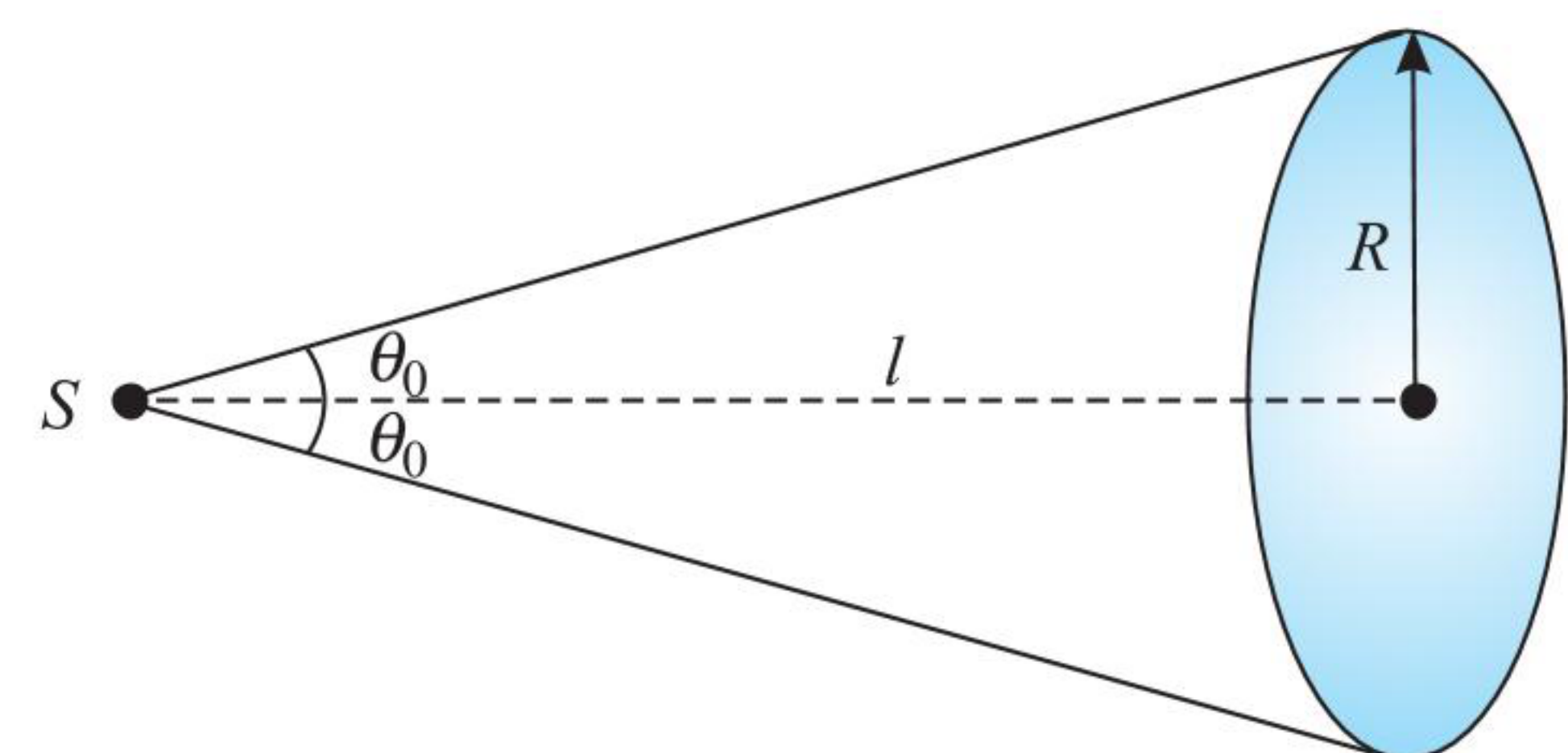
$$y = l \tan \theta \Rightarrow dy = l \sec^2 \theta d\theta$$

$$dF = \frac{2I}{c} 2\pi l \tan \theta l \sec^2 \theta d\theta \cos^2 \theta \left[\because \frac{l^2}{r^2} = \cos^2 \theta \right]$$

$$= 2 \left(\frac{P}{c 4\pi r^2} \right) 2\pi l^2 \tan \theta d\theta$$

$$dF = \frac{P}{c} \sin \theta \cos \theta d\theta$$

$$F = \int_0^{\theta_0} \frac{P}{c} \sin \theta \cos \theta d\theta = \frac{P}{2c} \sin^2 \theta_0$$

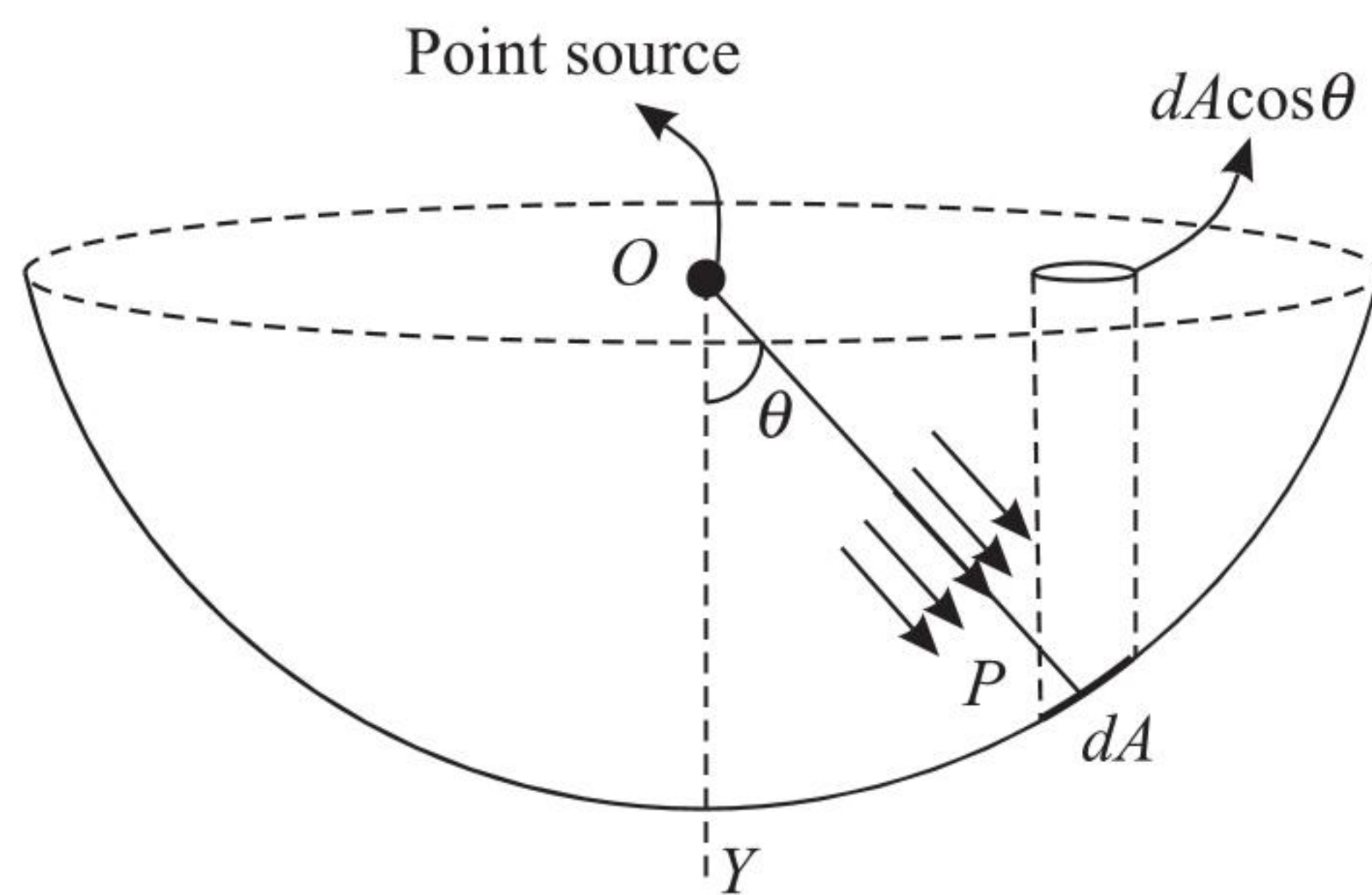


$$\Rightarrow F = \frac{P}{2c} \frac{R^2}{R^2 + l^2} = \frac{P}{2c} \left[\frac{1}{1 + (l/R)^2} \right] = \frac{P}{2c} \left(\frac{1}{1 + \eta^2} \right)$$

ILLUSTRATION 5.11

A point source of light is placed at the centre of curvature of a hemispherical surface. The radius of curvature is r and the inner surface is completely reflecting. Find the force on the hemisphere due to the light falling on it if the source emits a power W .

Sol. The intensity of the light at the hemisphere surface will be uniform and should be equal to, $I = \frac{W}{(4\pi r^2)}$.



Let us take a small elemental area dA at the point P of the hemisphere. The power incident on this elemental area

$$dW = I \cdot dA = \left(\frac{W}{4\pi r^2} \right) dA$$

The corresponding momentum incident on this elemental area per unit time is $\frac{dp}{dt} = \frac{dW}{c} = \frac{W dA}{4\pi r^2 c}$

As the light is reflected back, the change in momentum per unit time, i.e., the force on dA is

$$dF = \frac{2W dA}{4\pi r^2 c}$$

Suppose the radius OP through the area dA makes an angle θ with the symmetry axis OY . The force on dA is along this radius. By symmetry, the resultant force on the hemisphere is along OY . The component of dF along OY is

$$dF \cos \theta = \frac{2W dA}{4\pi r^2 c} \cos \theta$$

If we project the area dA on the plane containing the rim, the projection is $dA \cos \theta$. Thus, the component of dF along OY is

$$dF \cos \theta = \frac{2W}{4\pi r^2 c} (\text{projection of } dA)$$

The net force along OY is $F = \frac{2W}{4\pi r^2 c} (\text{projection of } dA)$

When all the small areas dA are projected, we get the area enclosed by the rim which is πr^2 .

$$\text{Thus, } F = \frac{2W}{4\pi r^2 c} \times \pi r^2 = \frac{W}{2c}$$

ILLUSTRATION 5.12

In the path of a uniform light beam of large cross-sectional area and intensity I , a solid sphere of radius R which is perfectly reflecting is placed. Find the force exerted on this sphere due to the light beam.

Sol. The front surface of the sphere will be illuminated by the light as shown in figure, as angle of incidence of

light is different at different position of the front face of the sphere.

We can consider a small elemental circular strip (ring) of angular width $d\theta$ on its surface at an angle θ from its horizontal diameter as shown. Let the area of this elemental strip is dS , then

$$dS = 2\pi R \sin \theta R d\theta \quad \dots(i)$$

$R \sin \theta$ is the radius of this circular strip (ring).

We take the projection of area of this slant elemental circular strip in vertical plane. Let dA be the projection of the slant strip area dS along the cross-sectional plane of the light beam and it is given as

$$dA = dS \cos \theta \quad \dots(ii)$$

Hence, the power of light incident on this strip is $dP = IdA$.

The momentum of photons per second incident on this strip is

$$dp = \frac{dP}{c} = \frac{IdA}{c} \quad \dots(iii)$$

These photons are incident at an angle θ to the normal N of this strip. As the surface of the sphere is perfectly reflecting, the photons will be reflected at the same angle θ to N as shown in the figure.

Hence, the change in linear momentum of photons is along the normal and thus force exerted on this strip along the normal is

$$dF = 2dp \cos \theta = \frac{2IdA}{c} \cos \theta \quad \dots(iv)$$

Thus, the net force on sphere will be along the x -axis only.

Thus, the force on the sphere will be given as

$$\begin{aligned} F &= \int dF \cos \theta = \int \frac{2IdA}{c} \cos^2 \theta \\ &= \int_0^{\pi/2} \frac{2I}{c} (2\pi R \sin \theta \cos \theta R d\theta) \cos^2 \theta \\ &= \frac{4I\pi R^2}{c} \int_0^{\pi/2} \cos^3 \theta \sin \theta d\theta \\ &= \frac{4I\pi R^2}{c} \left[-\frac{\cos^4 \theta}{4} \right]_0^{\pi/2} \\ &= \frac{I\pi R^2}{c} [1 - 0] = \frac{I\pi R^2}{c} \quad \dots(v) \end{aligned}$$

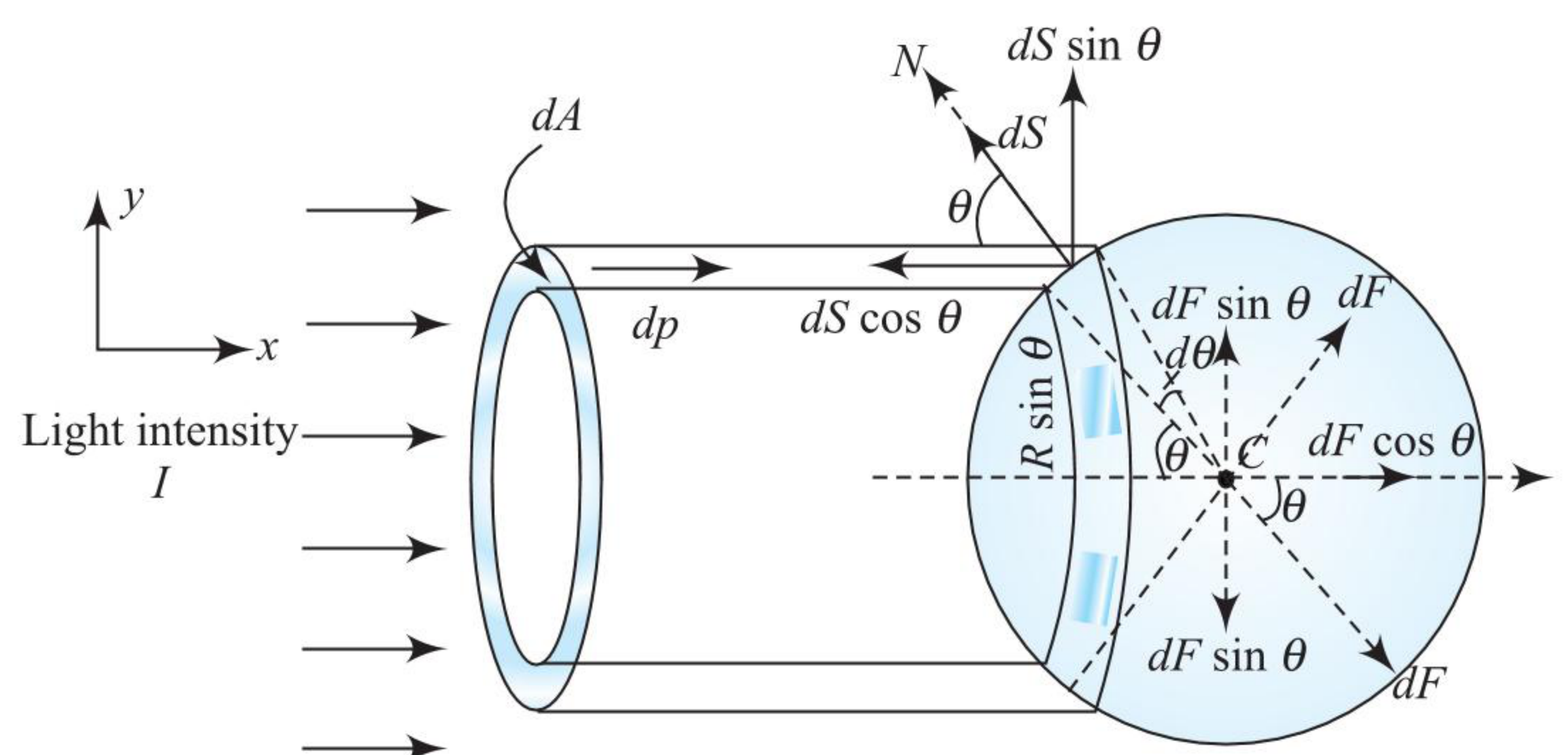
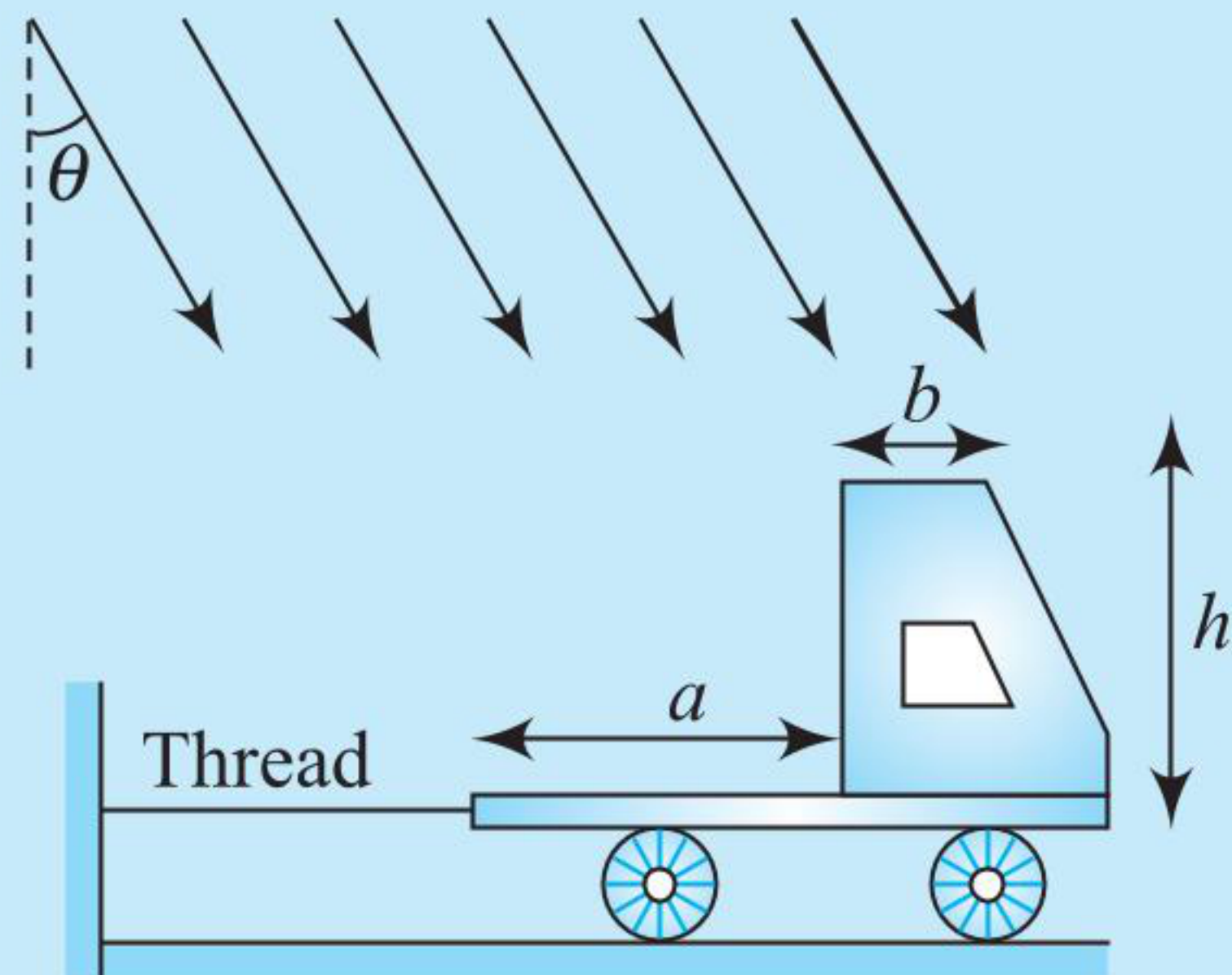


ILLUSTRATION 5.13

A toy truck has dimensions as shown in figure and its width, normal to the plane of this paper, is d . The sun rays are incident on it as shown in the figure. If intensity of rays is I and all surfaces of truck are perfectly black, calculate tension in the thread used to keep the truck stationary. Neglect friction.



Sol. First, we have to calculate power incident on the truck and then momentum of light photons incident per second. Since the truck surfaces are perfectly black, therefore momentum finally reduces to zero. It means that the rate of change of momentum of light is equal to the momentum of photons incident per second.

Area of top of the driving cabin = bd .

Light is incident due to the component $I \cos \theta$ of intensity I .

Power incident on top of the cabin = $bdI \cos \theta$

Similarly, power incident on rear horizontal part of the truck is $adI \cos \theta$ and power incident on rear vertical wall of the driving cabin is $hdI \sin \theta$.

Total power incident on the truck is

$$P = (b \cos \theta + a \cos \theta + h \sin \theta) Id$$

Momentum of these photons is $p = P/c$ (where c is speed of light in vacuum).

But rate of change in momentum of photons,

$$p = (Id/c)(b \cos \theta + a \cos \theta + h \sin \theta)$$

This momentum is inclined at angle θ with the vertical. Its vertical component $p \cos \theta$ is balanced by normal reaction of the floor and horizontal component $p \sin \theta$ is balanced by tension in the thread.

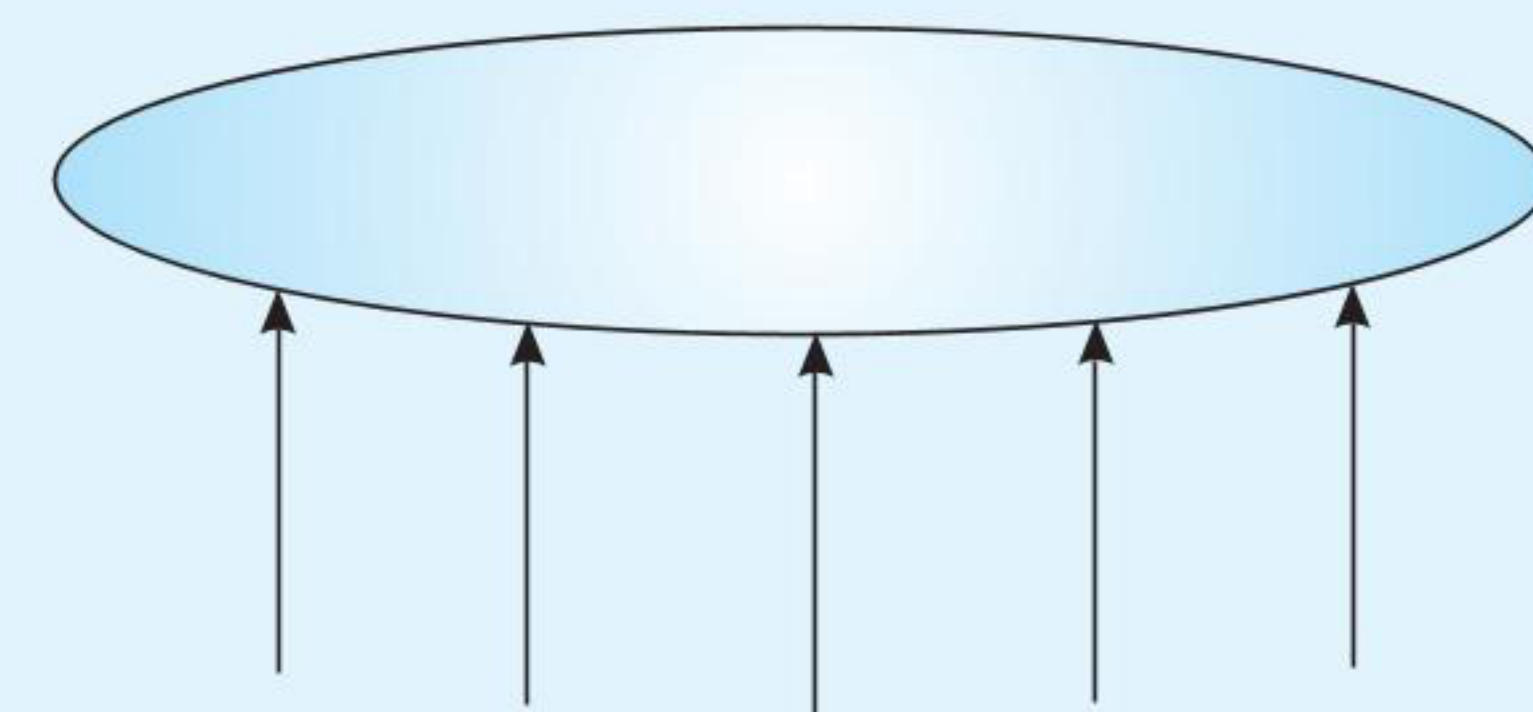
$$\text{Tension} = p \sin \theta = (Id/c)(b \cos \theta + a \cos \theta + h \sin \theta) \sin \theta$$

CONCEPT APPLICATION EXERCISE 5.1

- The intensity of direct sunlight before it passes through the earth's atmosphere is 1.4 kWm^{-2} . If it is completely absorbed, find the corresponding radiation pressure.
- Monochromatic light of wavelength 632.8 nm is produced by a helium-neon laser. The power emitted is 9.42 mW .
 - Find the energy and momentum of each photon in the light beam.
 - How many photons per second, on an average, arrive at a target irradiated by this beam? (Assume the beam to have uniform cross section, which is less than the target area.)
 - How fast does a hydrogen atom have to travel in order to have the same momentum as that of the photon? Given that mass of hydrogen atom is $1.66 \times 10^{-27} \text{ kg}$.

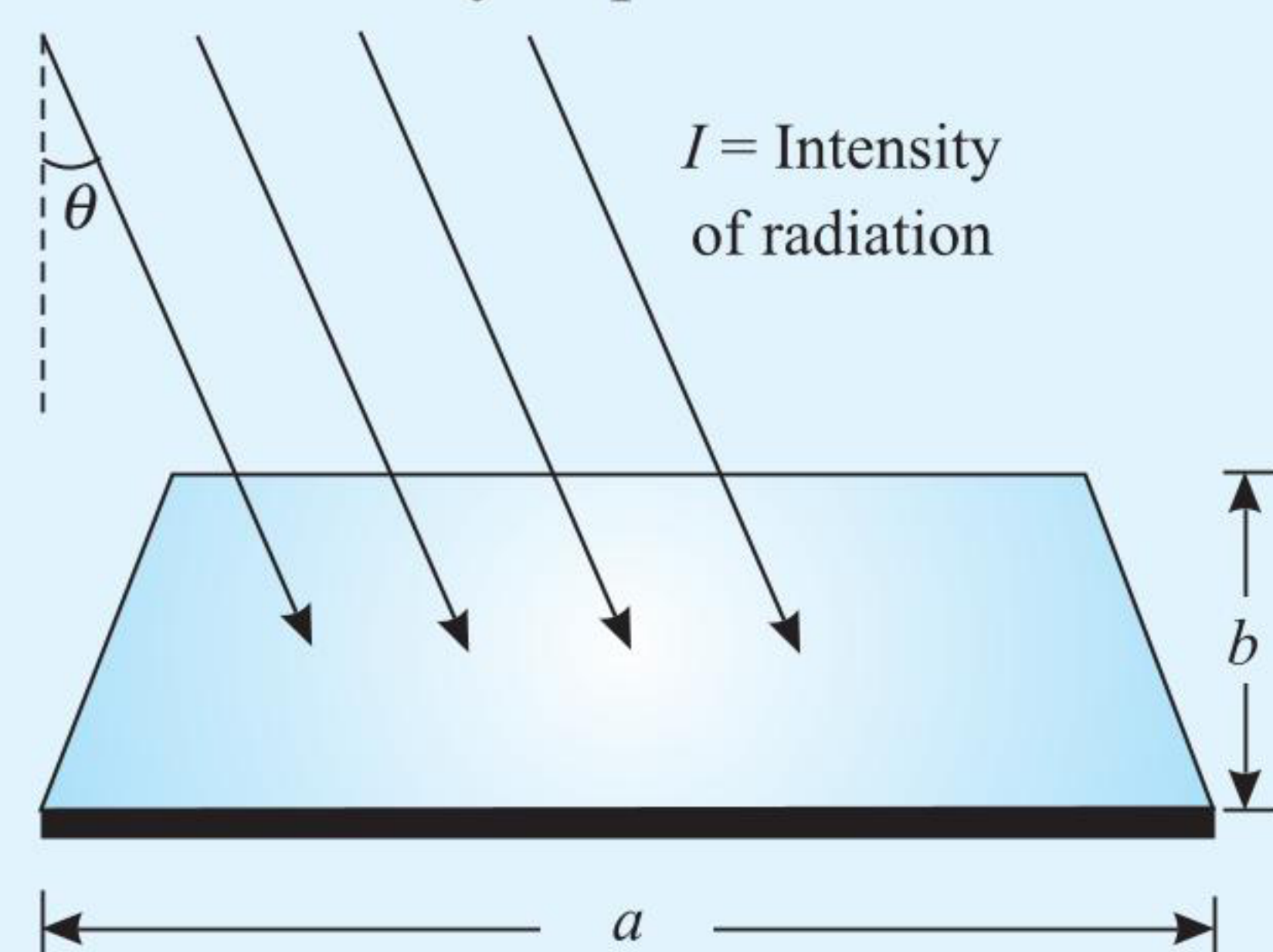
- An electron and a photon each has a wavelength of 1.0 nm . Find (a) their momenta, (b) the energy of the photon, and (c) the kinetic energy of electron. Take $h = 6.63 \times 10^{-34} \text{ J s}$.

- A plate is kept in front of a beam of photons. The plate reflects 40% of the incident photons and absorbs the remainder. If the plate absorbs energy at a rate 1200 Js^{-1} , find the net force acting on it.
- A sensor is exposed for 0.1 s to a 200 W lamp 10 m away. The sensor has an opening that is 20 mm in diameter. How many photons enter the sensor if the wavelength of light is 600 nm ? Assume that all the energy of the lamp is given off as light.
- Photons of wavelength $\lambda = 662 \text{ nm}$ are incident normally on a perfectly reflecting screen. Calculate their number per second falling on the screen such that the exerted force is 1 N .
- The sun delivers about 1.4 kW m^{-2} of electromagnetic flux to the earth's surface. Calculate:
 - the total power incident on a roof of dimensions $8 \text{ m} \times 20 \text{ m}$.
 - the solar energy in joules incident on the roof in 1 h .
 - the radiation pressure and force assuming the roof to be a perfect absorber.
- A plate of mass 10 g is in equilibrium in air due to the force exerted by a light beam on the plate. Calculate power of the beam. Assume that the plate is perfectly absorbing.



- The sun gives light at the rate of 1400 Wm^{-2} of area perpendicular to the direction of light. Assume λ (sunlight) = $6000 \text{ \AA}$. Calculate
 - the number of photons per second arriving at 1 m^2 area at earth.
 - the number of photons emitted from the sun per second assuming the average radius of the Earth's orbit is $1.49 \times 10^{11} \text{ m}$.
- A radiation of wavelength 200 nm is propagating in the form of a parallel surface. The intensity of the beam is 5 mW and its cross-sectional area is 1.0 mm^2 . Find the pressure exerted by radiation on the metallic surface if the radiation is completely reflected.
- A laser emits a light pulse of duration $T = 0.10 \text{ ms}$ and energy $E = 10 \text{ J}$. Find the mean pressure exerted by such a pulse when it is focused on a spot of diameter $d = 10 \text{ }\mu\text{m}$ on a surface perpendicular to the beam and with a reflection coefficient $\rho = 0.5$.
- A plane light wave of intensity $I = 0.50 \text{ W cm}^{-2}$ falls on a plane mirror of reflection coefficient $\rho = 0.8$. The angle of incidence $\theta = 45^\circ$. Find the normal pressure exerted by light on that surface.
- A plank of mass m is lying on a rough surface having coefficient of friction as μ in situation as shown in

figure. Find the acceleration of the plank assuming that it slips and the surface of body exposed to radiation is black body.



ANSWERS

1. $4.66 \times 10^{-6} \text{ Nm}^{-2}$
2. (a) 1.963 eV; $1.046 \times 10^{-27} \text{ kg m s}^{-1}$ (b) 3×10^{16} (c) 0.63 m s^{-1}
3. (a) $6.63 \times 10^{-25} \text{ kg m s}^{-1}$ (b) 1243.1 eV (c) 1.51 eV
4. $1.86 \times 10^{-5} \text{ N}$ 5. 1.53×10^{13} 6. 5×10^{26}
7. (a) 224 kW (b) 806.4 MJ (c) $4.7 \times 10^{-6} \text{ Nm}^{-2}$; $7.52 \times 10^{-4} \text{ N}$
8. $3 \times 10^{-7} \text{ W}$ 9. (a) $n = 4.22 \times 10^{21}$ (b) 1.178×10^{45}
10. $3.33 \times 10^{-5} \text{ Nm}^{-2}$ 11. $6.37 \times 10^6 \text{ Nm}^{-2}$
12. $1.25 \times 10^{-5} \text{ Nm}^{-2}$ 13. $\frac{F \sin \theta - \mu(mg + F \cos \theta)}{m}$

ELECTRON EMISSION

We know that the electrons in the outermost orbit of an atom are at maximum distance from the nucleus and hence most loosely bound to it. These are called free electrons. The electrons in metals which are free to move within the volume of metal are called free electrons. These electrons do not get ejected out of the surface of metal on their own. This is due to the reason that whenever an electron tries to leave the surface, the surface acquires a positive charge which pulls back the electron. To escape from the surface, an electron has to do a definite amount of work to overcome the force restraining it. To do this work, the electron must possess a certain minimum energy which can be imparted to it by an external source. This minimum energy is called the **work function** of the metal and is denoted by ϕ_0 .

The minimum energy that must be supplied to liberate the most weakly bound surface electrons from a metal without giving them any velocity is called the **work function** of the metal.

Work function is measured in electron volt (eV) where

$$1\text{eV} = 1\text{e} \times 1\text{V} = (1.6 \times 10^{-19} \text{ C})(1\text{V}) = 1.6 \times 10^{-19} \text{ J}$$

Work functions of some photosensitive metals

Metal	Work function (eV)	Metal	Work function (eV)
Cesium	1.9	Calcium	3.2
Potassium	2.2	Copper	4.5
Sodium	2.3	Silver	4.7
Lithium	2.5	Platinum	5.6

When a free electron gets extra energy ($\geq$ work function) from an external agent, it is able to overcome potential barrier and is ejected out. There are a number of ways in which energy from outside can be supplied. Hence, there are different ways in which electron emission can take place. These ways are listed below:

Photoelectric emission: When electromagnetic radiations of suitable wavelength (or frequency) are incident on a metallic surface, then electrons are emitted, this phenomenon is called photoelectric effect.

Thermionic emission: In this case, extra energy is given to electrons to overcome potential barrier in the form of heat by passing current through a filament (Joule's heating effect).

Field emission: In this case, metal is placed in a strong electric field due to which the electrons are accelerated to such a speed that the corresponding kinetic energy is sufficient to overcome potential barrier.

Secondary emission: Electrons from metal can also be ejected by impinging a beam of high-speed electrons so that through collision, electrons acquire sufficient kinetic energy to overcome potential barrier. This type of electron emission is called secondary emission.

PHOTOELECTRIC EFFECT

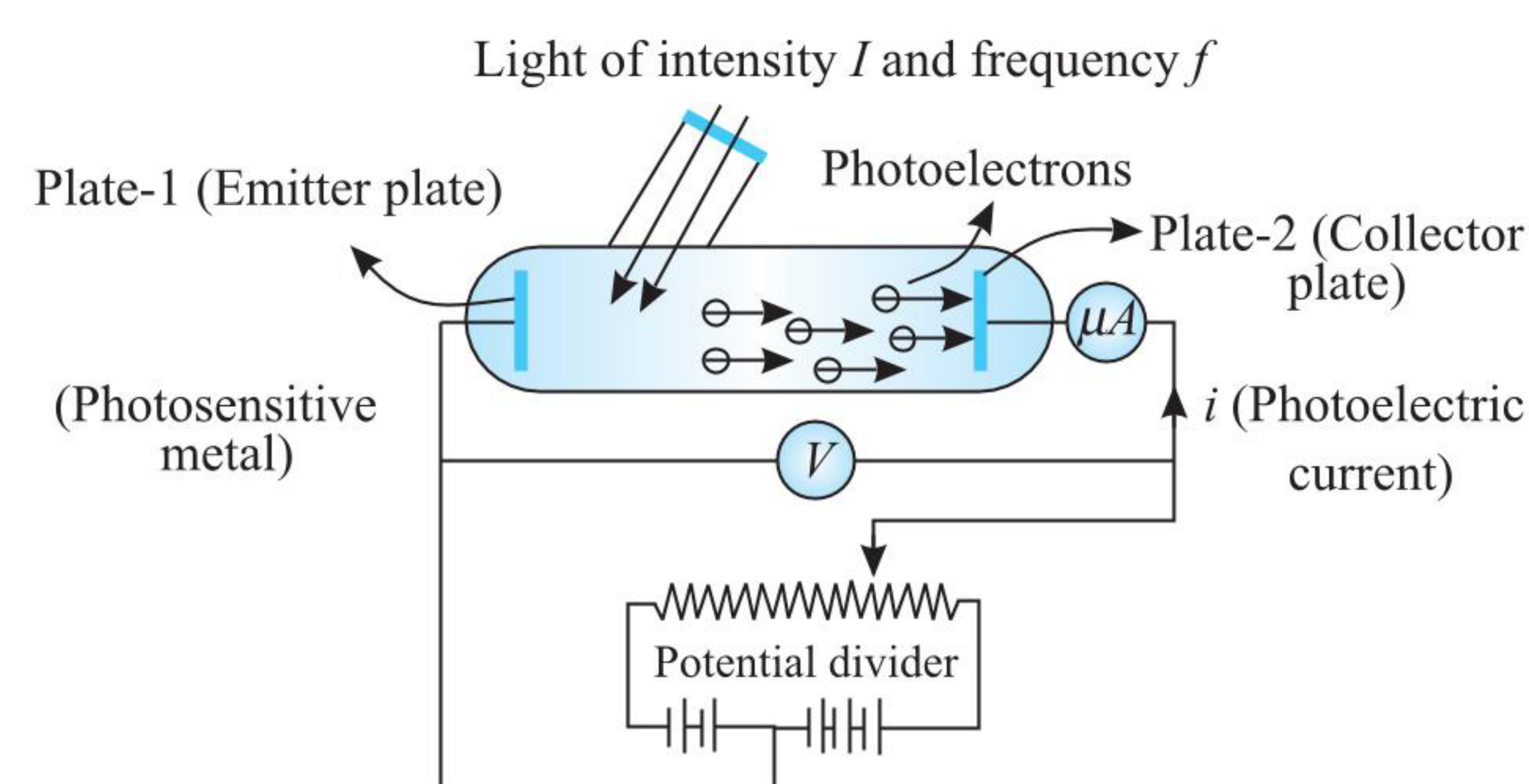
The discovery of photoelectric effect is due to the extensive investigations carried out by W. Smith, Heinrich Hertz, Wilhelm Hallwachs, Elster, Geital, J.J. Thomson and P. Lenard.

When electromagnetic radiations of suitable wavelength (or frequency) are incident on a metallic surface then electrons are emitted, this phenomenon is called **photoelectric effect**. The electrons so emitted are known as **photoelectrons**. And the current so obtained is known as **photoelectric current**.

Alkali metals Li, Na, K, Cs, etc., show photoelectric emission with visible light. Zn, Cd, Mg, etc., show photoelectric emission with ultraviolet light.

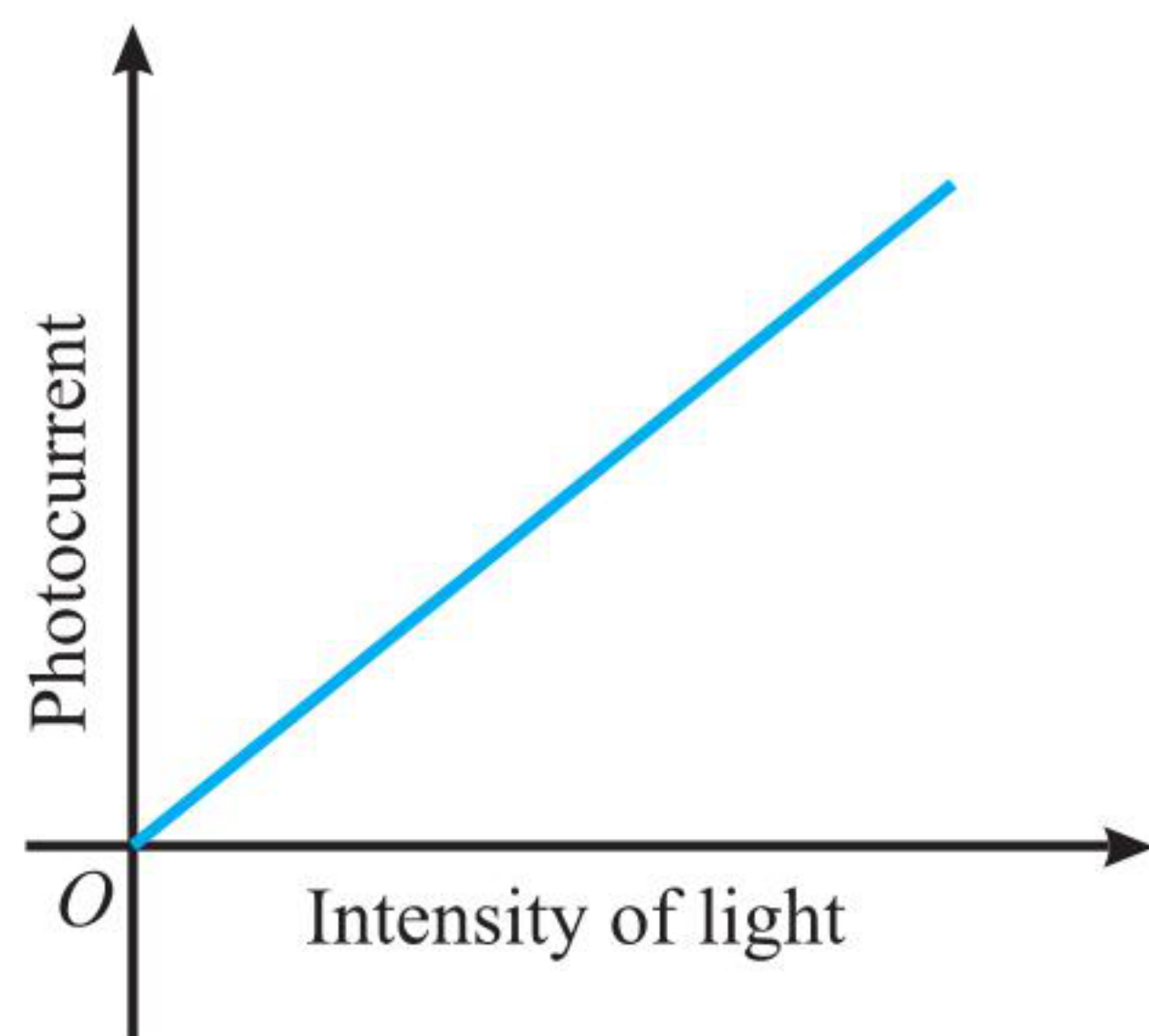
Study of Photoelectric Effect

Figure shows the experimental setup to study the photoelectric effect.

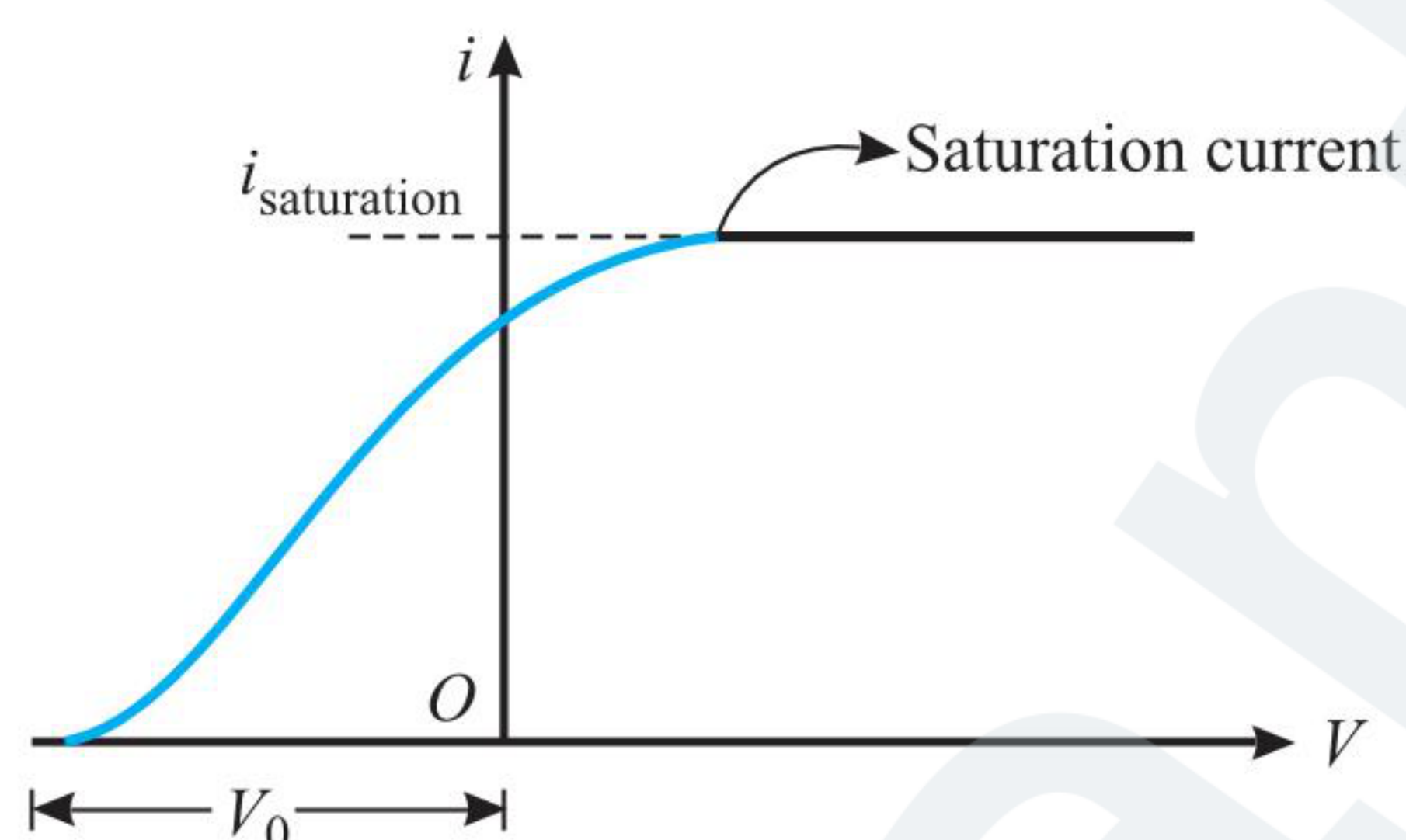


When monochromatic light of sufficiently short wavelength falls on the plate-1 called cathode which is made of some photosensitive material, electrons are emitted from it. The electrons are collected by the plate-2 called anode and a current flow in the circuit. Both these electrodes (cathode and anode) are enclosed in an evacuated glass tube which is provided with a quartz window W . Quartz is used because we have to use UV light to which glass is opaque. The anode can be maintained at a suitable positive or negative potential with the help of a potential divider. The potential difference between the electrodes is measured with the help of voltmeter. When light of suitable wavelength falls on cathode, electrons are emitted. When anode is positive w.r.t. cathode, electrons are attracted to it and the resulting photoelectric current (i) is measured with a microammeter.

Experiment I. Effect of intensity of light on photoelectric current: By keeping the frequency of light constant, we study the variation of photoelectric current (i) with the intensity of light (I). The anode is kept at a fixed positive potential (V) w.r.t. the cathode. If the intensity of radiation (I) is varied (or more number of photons are allowed to fall per unit time), a graph between intensity of light and photoelectric current (PEC) is a straight line passing through the origin as shown in figure. The photoelectric current is directly proportional to the intensity of incident radiation.



Experiment II. Effect of potential on photoelectric current: By keeping the intensity and frequency of incident light constant, we study the variation of photoelectric current with potential difference between cathode and anode. The shape of graph between intensity (I) and frequency (f) is as shown in the figure. The portion of the graph towards the right of the current (i) axis shows that as the collector plate (anode) is made even slightly positive, almost all the electron ejected by the emitter plate (cathode) are collected by anode and soon the current attains its maximum value. The maximum value of the photoelectric current is called the **saturation current**.



The portion of the graph towards left of the current axis shows the effect of making collector plate negative with respect to emitter plate. Since the current does not immediately drop to zero, it is obvious that the photoelectrons are emitted with velocities ranging from zero to a certain maximum. When we give positive potential given to collector plate, we call it as accelerating potential, and if we give negative potential given to collector plate, we call it retarding potential.

The retarding potential (given to the collector) for which the photoelectric current becomes zero is called the **cut-off or stopping potential** (V_0) for the corresponding frequency of the incident light.

If m is the mass of the electron and $v_{\max}$ is the maximum velocity of the most energetic ejected electron,

$$\frac{1}{2}mv_{\max}^2 = eV_0 \quad \dots(i)$$

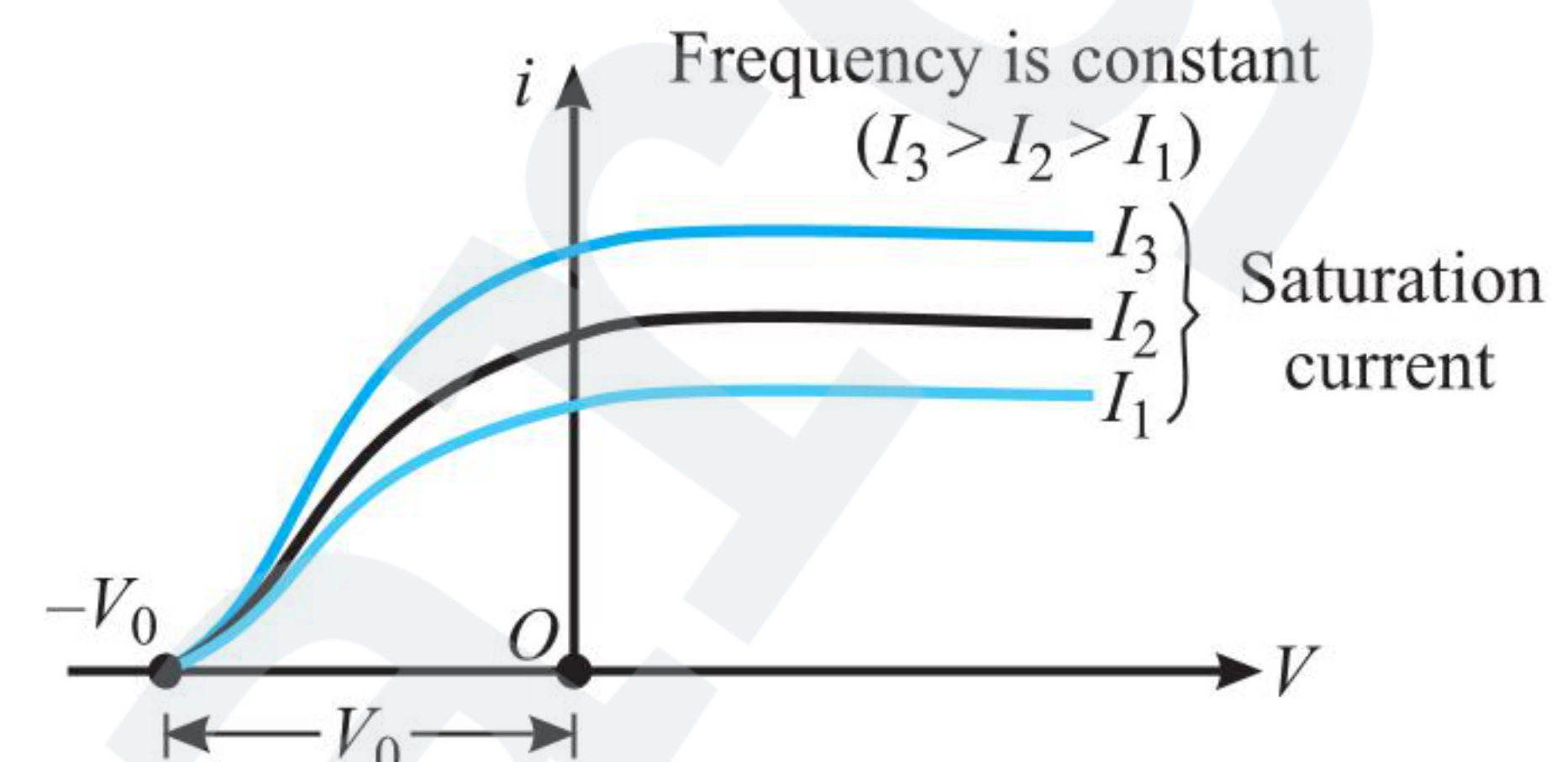
If $K_{\max}$ is the corresponding kinetic energy,

$$K_{\max} = \frac{1}{2}mv_{\max}^2 \quad \dots(ii)$$

Thus, from Eqs. (i) and (ii),

$$K_{\max} = eV_0 \quad \dots(iii)$$

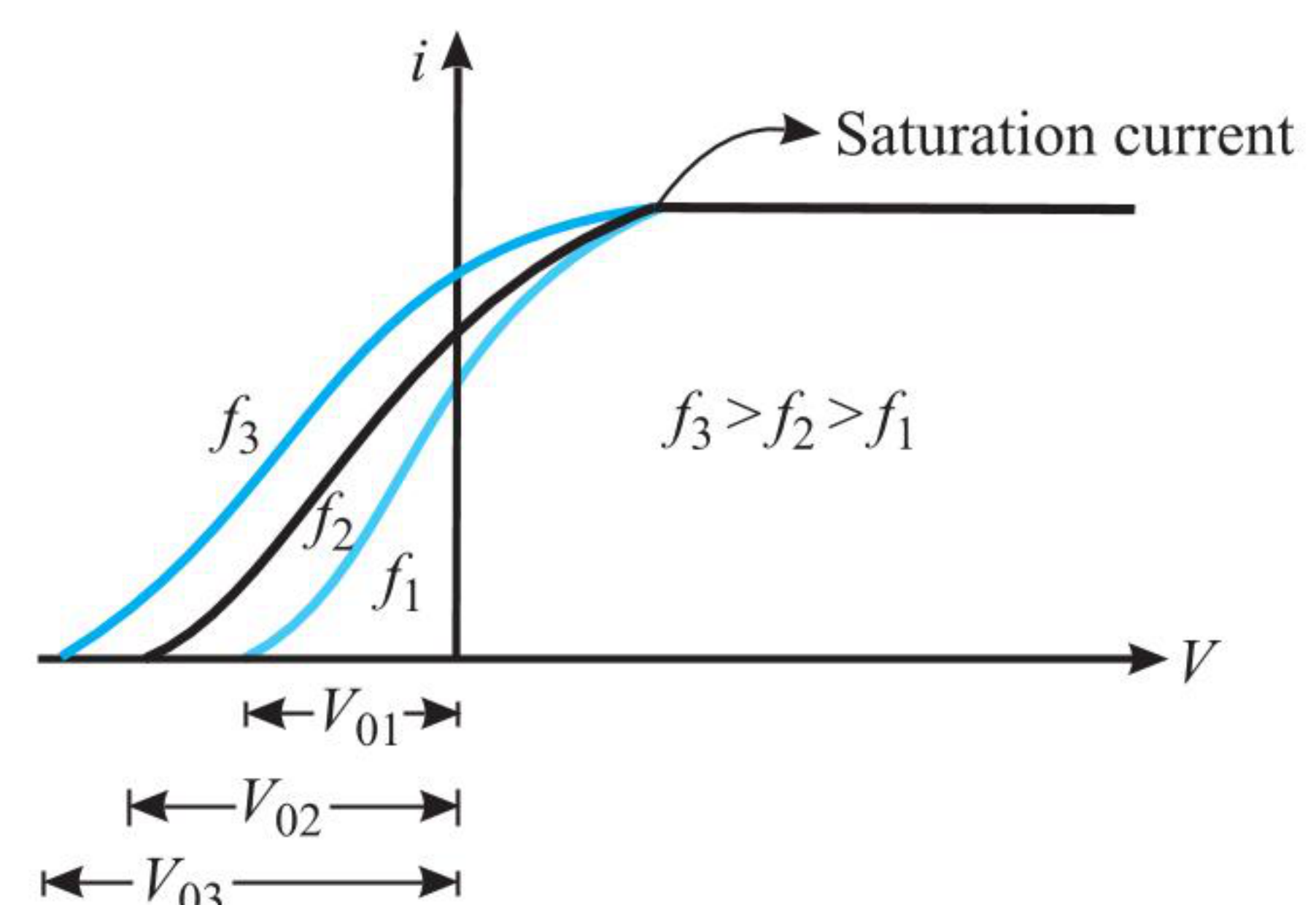
If the experiment is now repeated with incident lights of the same frequency (f) but of higher intensities I_2 and I_3 ($I_3 > I_2 > I_1$), it is found that stopping potential remains the same, i.e., V_0 , though the saturation current increases as shown in figure.



Important Point:

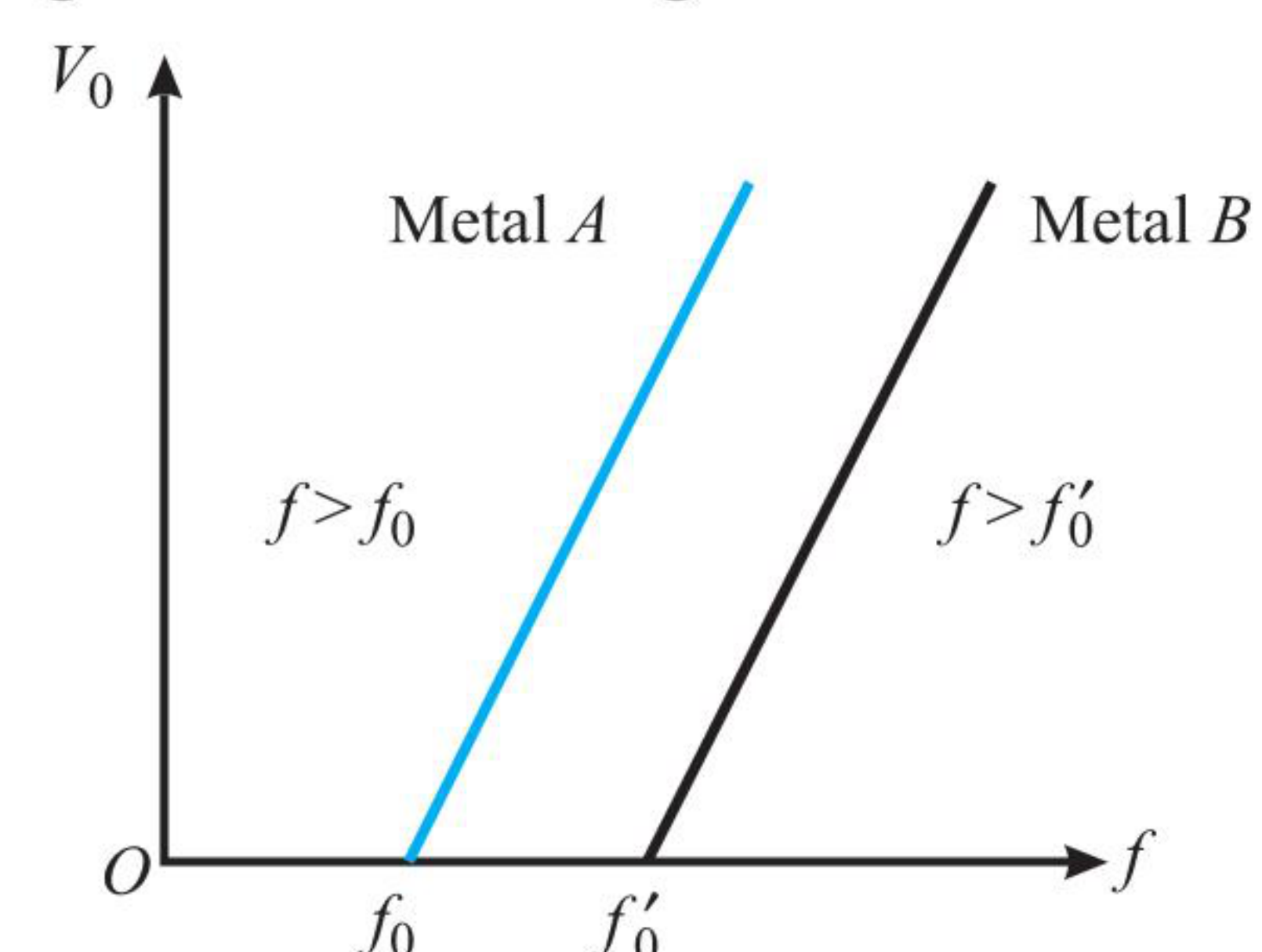
As stopping potential is a measure of $K_{\max}$, it is obvious that: Maximum kinetic energy of photoelectron ($K_{\max}$) does not depend on the intensity of light of a given frequency.

Experiment III. Effect of frequency of incident light on stopping potential: If we keep the intensity of light constant, we study the variation of photoelectric current with potential difference between emitter plate and collector plate. If light of different frequencies (keeping the intensity same) is used, then plots are obtained which are shown in figure.



We find that stopping potential for each frequency is different and $V_{03} > V_{02} > V_{01}$ (in magnitude). The higher the frequency of incident light, greater is the stopping potential. It means, the more is the energy of the photoelectrons ejected, i.e., $K_{\max}$ of ejected electrons increases with frequency of incident light.

The graph between frequency of radiation (f) and stopping potential (V_0) it is found to be **straight** line which is not passing through the origin as shown in figure.



Important Points:

- As stopping potential (V_0) varies linearly with frequency of radiation (f), $K_{\max}$ also varies linearly with frequency.
- The intercept of the graph on the frequency axis gives the frequency (f_0) that would just liberate electrons out of the emitter. If light of frequency less than f_0 is used, no electrons would be ejected.
- The minimum value of the frequency of light below which the photoelectric emission stops completely, however large may be the intensity of light, is called the **threshold frequency**.
- The number of photoelectrons emitted per second is proportional to the intensity of the incident light.
- There is a lower limit of frequency called threshold frequency below which no emission takes place, however high the intensity of the incident radiation may be.
- Above the threshold frequency, the maximum velocity with which electrons emerge is dependent solely on the frequency and not on the intensity of the incident light.
- The emission of photoelectrons is an instantaneous process. It means time difference between incidence of light and emission of photoelectrons is very small, may be even less than 10^{-9} s.

ILLUSTRATION 5.14

Will photoelectrons be emitted from a copper surface, of work function 4.4 eV, when illuminated by a visible light?

Sol. The threshold wavelength λ_0 corresponding to work function W is given by

$$\lambda_0 = \frac{hc}{W}$$

$$\text{or } \lambda_0 = \frac{12400}{4.4} = 2820 \text{ \AA}$$

Since λ_0 does not lie in the visible range (4000 Å to 7500 Å), therefore it cannot eject photoelectrons from copper.

ILLUSTRATION 5.15

Find the number of photons emitted per second by a 25 W source of monochromatic light of wavelength 6600 Å. What is the photoelectric current assuming 3% efficiency for photoelectric effect? Given $h = 6.6 \times 10^{-34}$ Js

Sol. Energy of each photon $= \frac{hc}{\lambda}$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{6600 \times 10^{-10}} = 3 \times 10^{-19} \text{ J}$$

Total energy emitted per second by 25 W source = 25 J

$$\text{Number of photons emitted per second} = \frac{25}{3 \times 10^{-19}} = 8.33 \times 10^{19}$$

Photoelectric current = 3% of photons emitted per second $\times$

$$\text{charge on electron} = \frac{3}{100} \times \frac{25}{3 \times 10^{-19}} \times 1.6 \times 10^{-19} = 0.4 \text{ \AA}$$

Failure of Wave Theory of Light in Explaining Photoelectric Effect**Intensity Problem**

Maxwell's electromagnetic wave theory of light says that the average intensity of light is directly proportional to the square of the electric field of the light wave. If we increase the intensity of light it results in increase in the electric field intensity. It pushes the surface electrons with more force. If more force acts in an electron it does more work, increasing the kinetic energy of the electron to greater extent. As a result, the emitted electrons will possess more kinetic energy if the intensity of incident radiation is more. However, this opposes the experimental observation that the maximum kinetic energy of photoelectrons does not depend on intensity of light.

Frequency Problem

When the radiation falls on the metallic surface as electromagnetic wave continuously and speed (distributed) uniformly, the electrons of the metallic surface will absorb the incident energy continuously sooner or later each electron will accumulate enough energy to overcome the potential barrier of the metal and escape from it. Hence photoelectric effect should occur at any frequency of incident light. Again this prediction of wave theory of light contradicts the experimental observation that the photoelectric effect can take place above a cutoff (threshold) frequency of incident light (not at all frequencies for a given a metal).

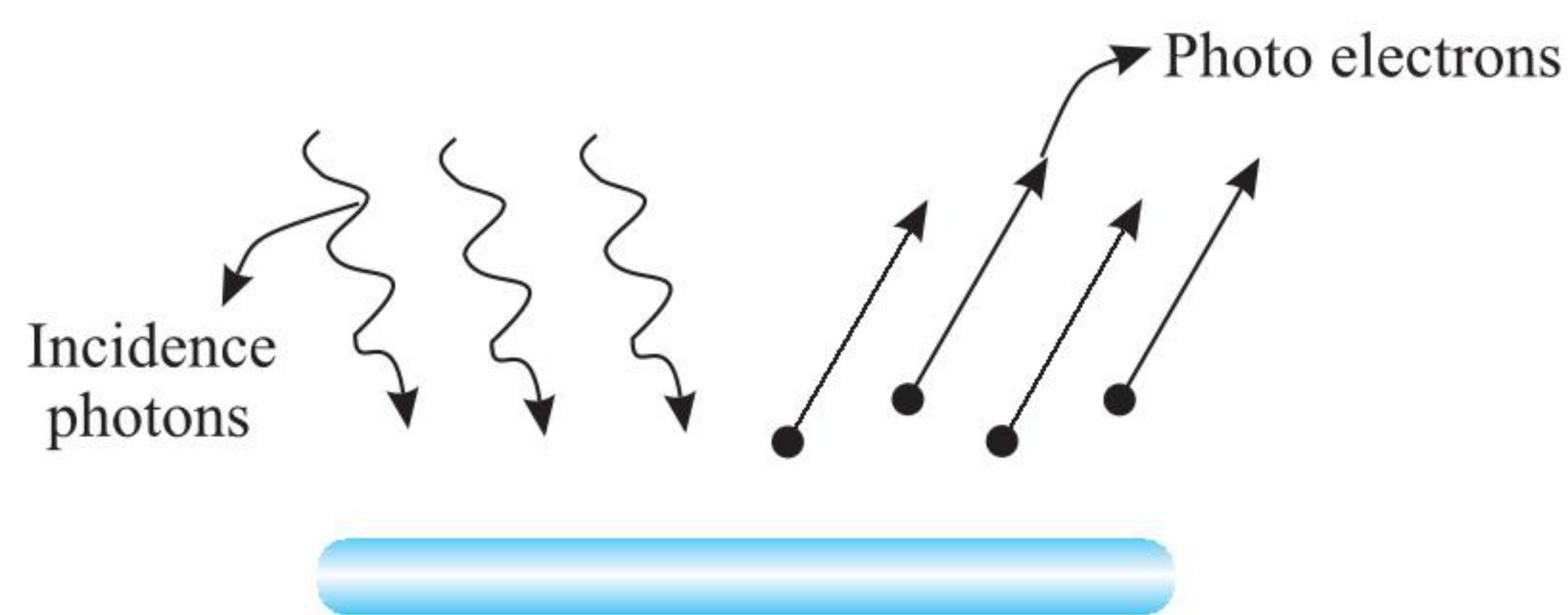
Time Delay Problem

If the radiation falls continuously and uniformly and the atoms will absorb the incident radiation continuously, each atom will receive radiation energy proportional to their cross-sectional area. Since the cross-section of an atom is very small, each atom will receive very little energy in each second. Therefore, it takes a long time for an atom to accumulate sufficient energy to eject an electron from it. However, this prediction also proved to be incorrect because observation says that the photoelectrons start coming out of the metal almost instantaneously after the light falls on it.

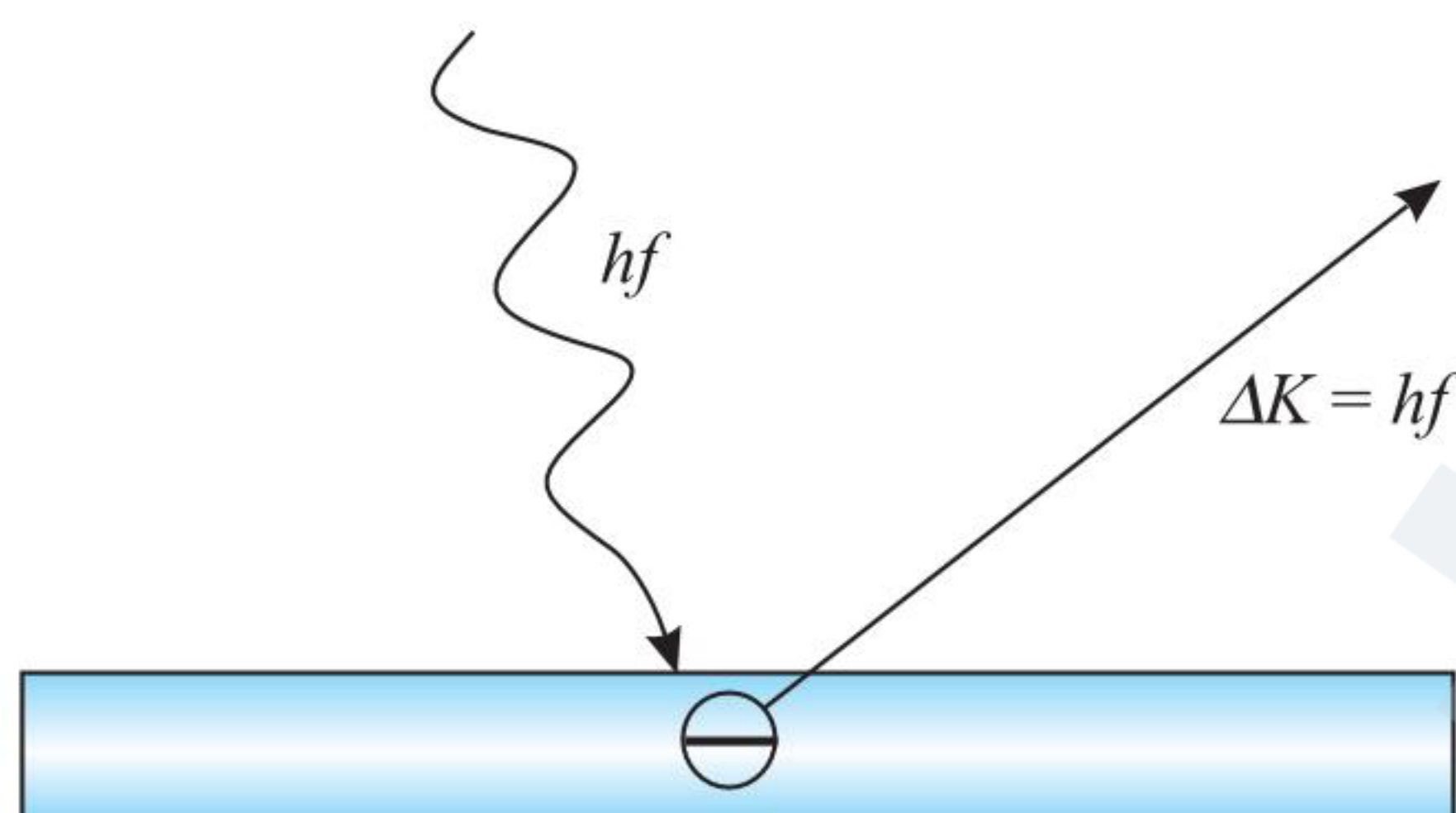
Einstein's Explanation for Photoelectric Effect

The most authentic explanation of photoelectric effect was given by Einstein. Einstein explained the Planck's concept of quantisation of electromagnetic waves more clearly. He assumed that if a radiation of frequency f propagating in vacuum, it can be considered a stream of photons. Each photon travels at speed of light and has energy hf . He put his theory for photoelectric effect as following way:

- The emission of photoelectron is instantaneous. The term instantaneous (or sudden) about the emission of electrons implies that "certain body" is colliding the electrons promptly. That "certain body" can be imagined as a pulse or packet of radiation. As radiation is an electromagnetic wave, we can imagine the light hitting the metal as electromagnetic wave pulse. Each pulse can be termed as photon (bundles or packet or quartz of radiation).



- (ii) When a photon interacts with an electron, it gives its entire energy to the electron and then exists no longer. It is absorbed by a single electron. The chance of two photons hitting the same electron simultaneously is practically zero as there is one-to-one interaction between the photons and the electrons.
- (iii) The emission of photoelectrons stops below certain frequency of incident radiation. This signifies that electron cannot absorb a fraction of an incident photon and cannot increase its kinetic energy by continuous absorption of photons one by one. In other words, each photon must be absorbed completely by an electron inside the metal.
- (iv) The maximum kinetic energy of (or stopping potential for) photoelectrons increases linearly (proportionality) with the frequency of the incident light. It means that the energy of each photon which is converted to (directly proportional to) the maximum kinetic energy of the photoelectrons must be varying linearly with the frequency of incident radiation. It can be shown that, the energy of a photon is



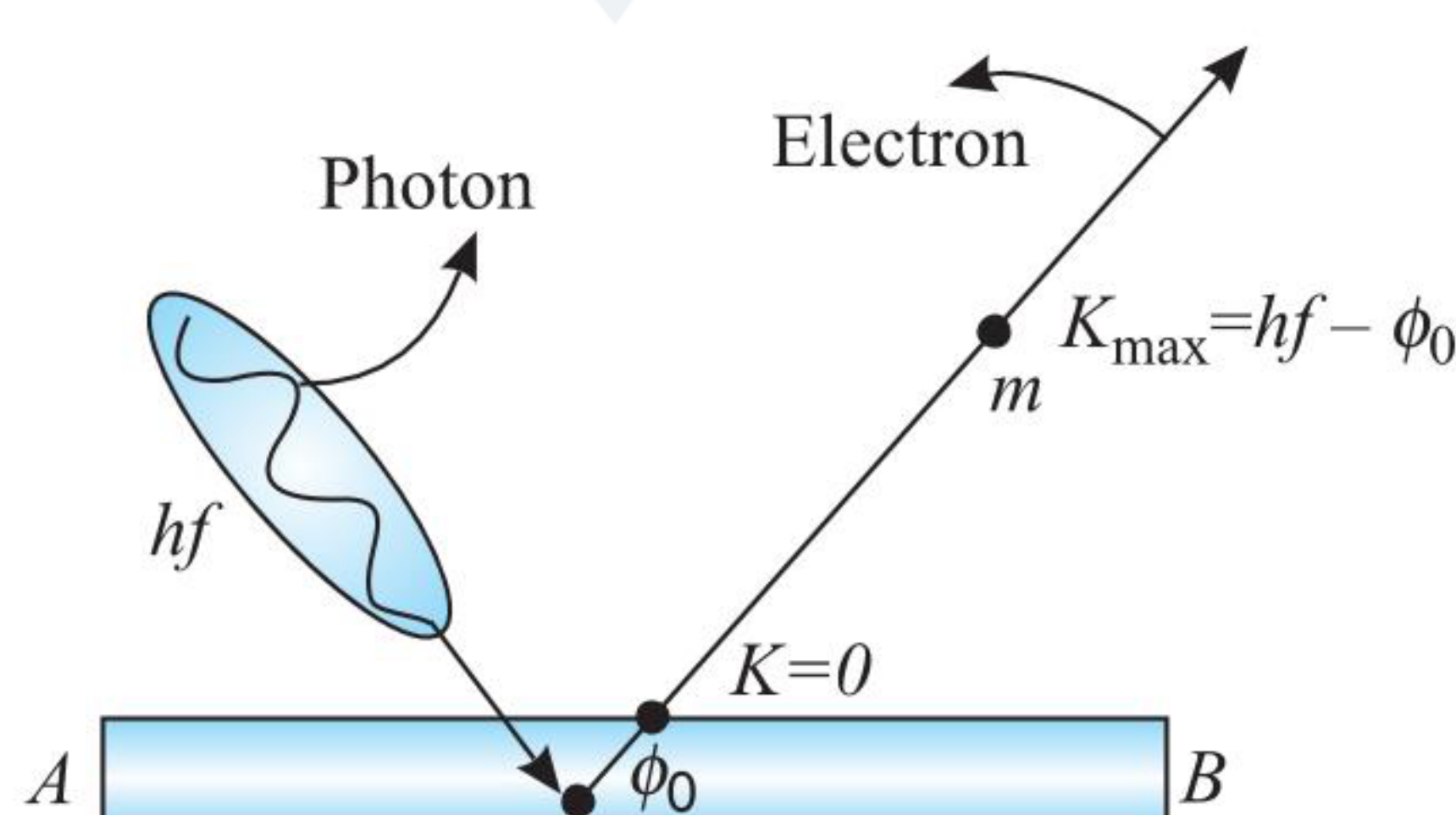
A photon kicks an electron increasing its KE by hf .

$$E_{\text{photon}} = hf$$

- (v) If the intensity of light increases, the photoelectric current increases proportionally. This means that each photon of a monochromatic light must carry equal energy, that is, hf . If the intensity of light increases, the number of photons striking the metal per second will increase. Eventually, more electrons will come out of the metal per second. In short, the rate of emission for photoelectrons is directly proportional to the rate of incident photons.

Einstein's Photoelectric Equation

According to Einstein, photon energy (hf) falling on a metal is utilized for two purposes:

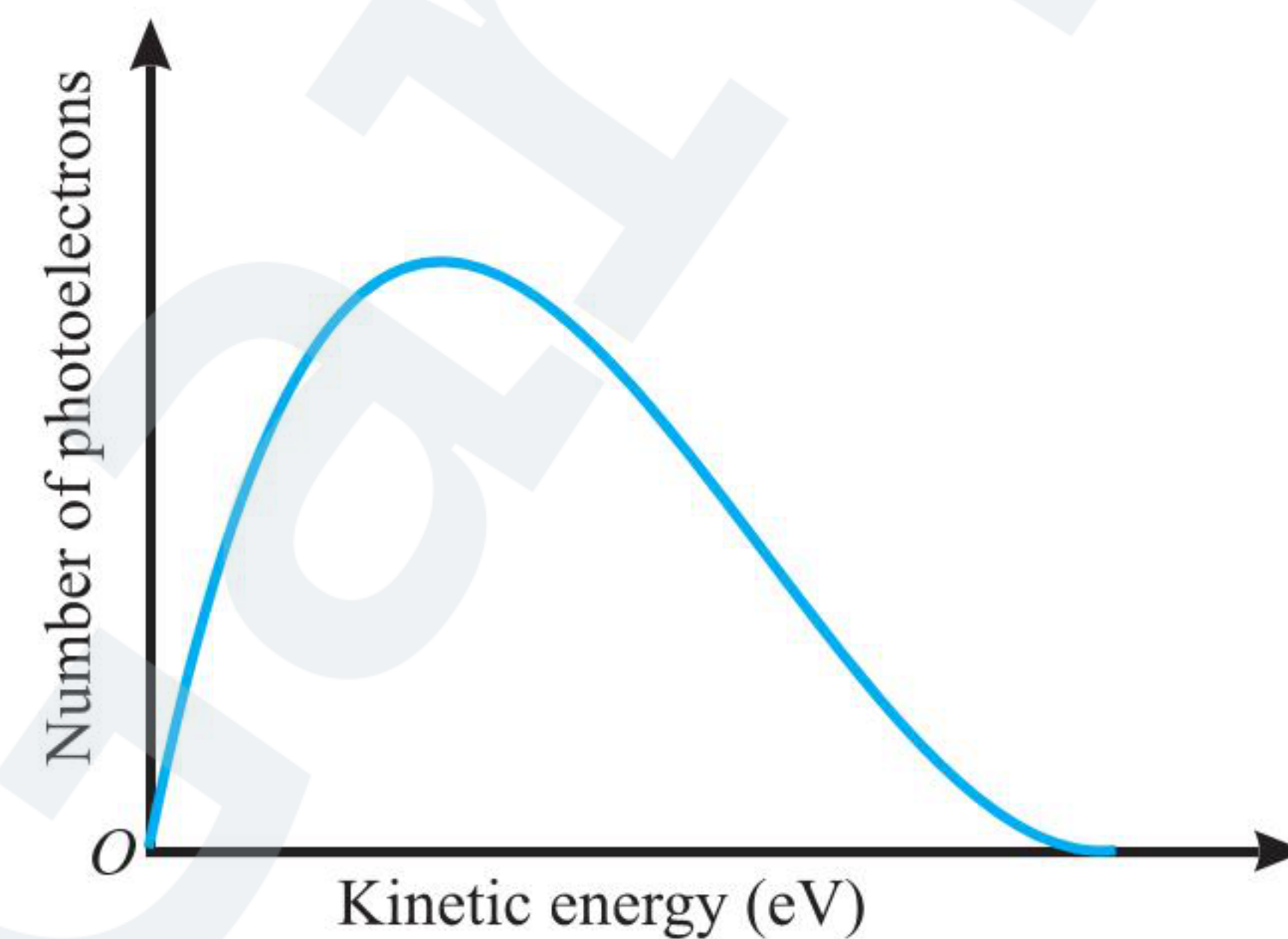


1. Partly for getting the electron free from the atom and away from the metal surface. This energy is known as the photoelectric work function of the metal and is represented by W_0 (or ϕ_0).
2. The balance of the photon energy is used up in giving the electron a kinetic energy, K_{max} being the maximum value of this kinetic energy.

So, we can write:

$$hf = W_0 + K_{\text{max}} \Rightarrow K_{\text{max}} = hf - W_0 \quad \dots(i)$$

Whenever photoelectric effect takes place, electrons are ejected out with kinetic energies ranging from 0 to K_{max} . The energy distribution of photoelectrons is as shown in figure.



Let us consider a photon which has got only that much energy which can just eject an electron from the metal surface. The frequency of such a photon is obviously the threshold frequency (f_0). Thus, when $f = f_0$, $K_{\text{max}} = 0$. From Eq. (i),

$$W_0 = hf_0 \quad \dots(ii)$$

Combining Eqs. (i) and (ii)

$$K_{\text{max}} = hf - hf_0 = h(f - f_0) = hc \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right) \quad \dots(iii)$$

Equation (iii) is called Einstein's photoelectric equation.

Important Point:

In Eq. (iii), if we write wavelengths, λ and λ_0 in $\text{\AA}$, the kinetic energy in electron-volts can be expressed as

$$K_{\text{max}} = 12400 \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right) \text{eV}$$

Finding Plank's Constant from Einstein's Photoelectric Equation

Robert Millikan verified Einstein's photoelectric equation experimentally and provided irrefutable evidence of the photon model of light.

According to Einstein's photoelectric equation,

$$K_{\text{max}} = hf - W_0 \quad \dots(i)$$

$$\text{Also, } K_{\text{max}} = eV_0 \quad \dots(ii)$$

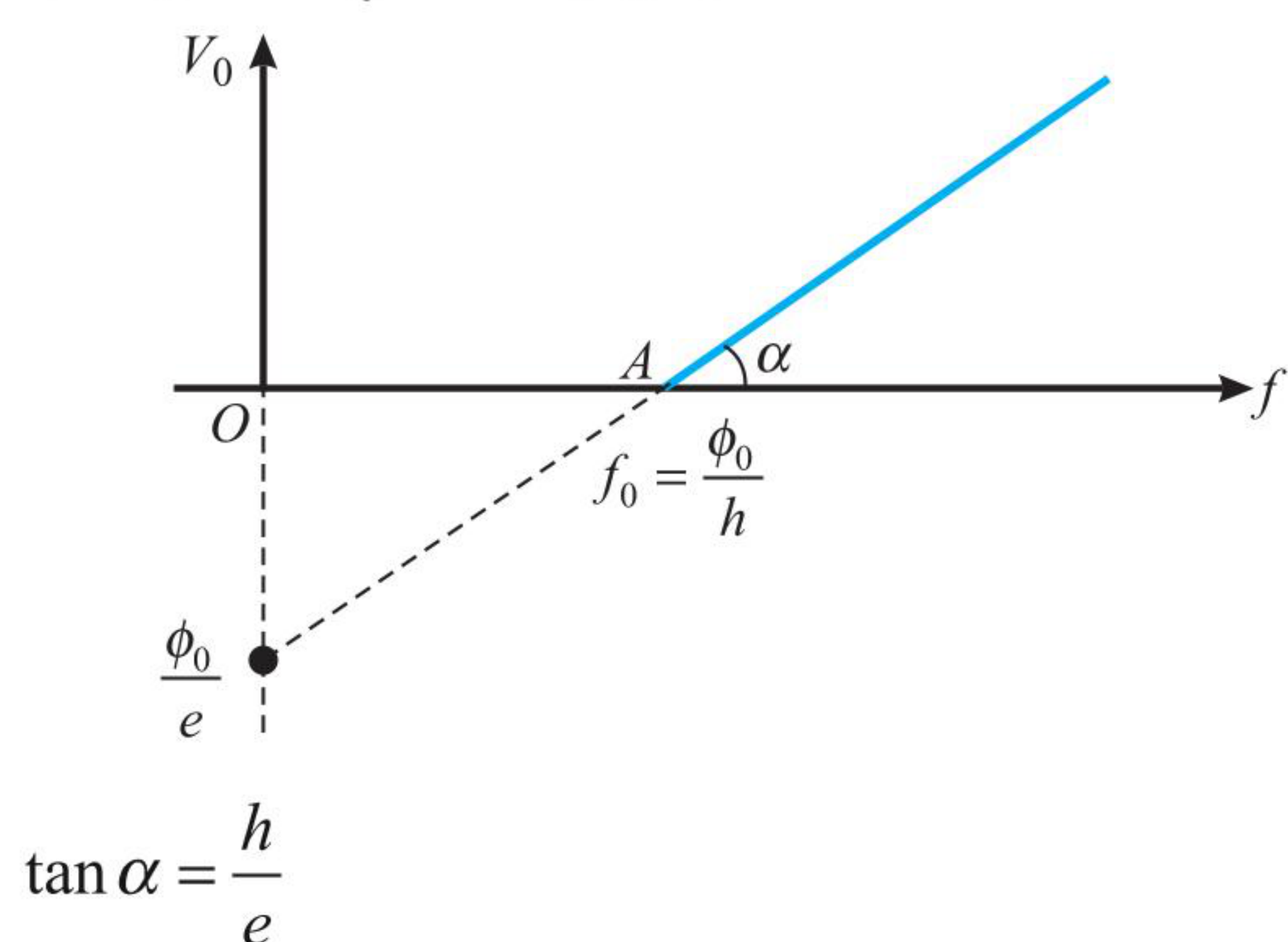
where V_0 is the stopping potential.

From Eqs. (i) and (ii),

$$eV_0 = hf - W_0 \quad \dots(iii)$$

$$\text{or } V_0 = \left(\frac{h}{e} \right) f - \frac{W_0}{e} \quad \dots(iv)$$

Equation (iv) implies that V_0 versus f curve is a straight line with slope (h/e) , which is independent of the nature of the material. Further, slope of V_0 versus f graph

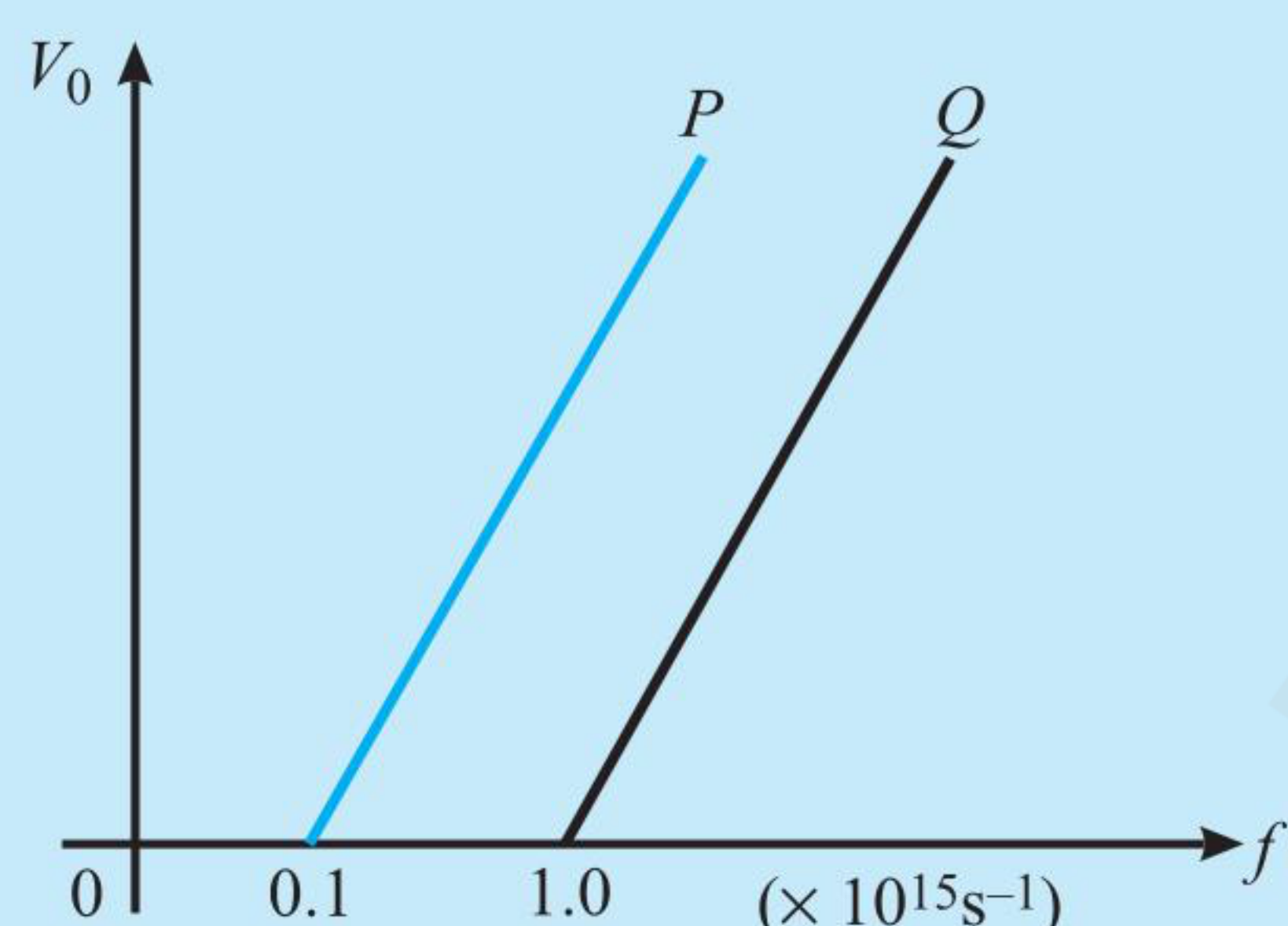


or $h = e \tan \alpha$... (v)

Measuring α and using the known value of e , Millikan calculated the value of h and the value so obtained agreed fairly well with that obtained from black body radiation experiments.

ILLUSTRATION 5.16

Figure shows the variation of stopping potential V_0 with the frequency f of the incident radiation for two photosensitive metals P and Q .



- Explain which metal has smaller threshold wavelength.
- Explain, giving reason, which metals emits photoelectrons having smaller kinetic energy, for the same wavelength of incident radiation.
- If the distance between the light source and metal P is doubled, how will the stopping potential change?

Sol.

- (a) Threshold wavelength, $\lambda_0 = \frac{c}{f_0}$

As $f_{0(Q)} > f_{0(P)}$

$\therefore \lambda_{0(Q)} < \lambda_{0(P)}$

Thus the metal Q has smaller threshold wavelength.

- (b) According to Einstein's photoelectric equation,

$$\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + \text{KE of photoelectron}$$

For the same λ of incident radiation, L.H.S. is constant. So metal Q with smaller value of λ_0 will emit photoelectrons of smaller KE

- (c) Stopping potential V_0 will remain same because it is independent of intensity and hence on distance between the light source and the metal surface.

ILLUSTRATION 5.17

The threshold frequency of a metal is f_0 . When the light of frequency $2f_0$ is incident on the metal plate, the maximum velocity of electrons emitted is v_1 . When the frequency of the incident radiation is increased to $5f_0$, the maximum velocity of electrons emitted is v_2 . Find the ratio of v_1 to v_2 .

Sol. The minimum amount of energy required to eject an electron from a metal surface (without imparting it any kinetic energy) is called work function of the metal. As f_0 is the threshold frequency, so $W = hf_0$

From Einstein's photoelectric equation, the maximum kinetic energy of emitted electron is given by

$$\frac{1}{2}mv_1^2 = h \times 2f_0 - W = h \times 2f_0 - hf_0 = hf_0$$

and $\frac{1}{2}mv_2^2 = h \times 5f_0 - hf_0 = 4hf_0$

$$\therefore \frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \frac{hf_0}{4hf_0}$$

or $\frac{v_1}{v_2} = \frac{1}{2} = 1:2$

ILLUSTRATION 5.18

Light is incident on the cathode of a photocell and the stopping voltages are measured for light of two different wavelengths. From the data given below, determine the work functions of the metal of the cathode in eV.

Wavelength (Å)	Stopping voltage (volt)
4000	0.9
4500	1.3

Sol. As $K_{\max} = hf - W_0$

or $eV_0 = \frac{hc}{\lambda} - W$

or $V_0 = \frac{hc}{e\lambda} - \frac{W_0}{e}$

$$\therefore \Delta V_0 = (V_0)_2 - (V_0)_1 = \left[\frac{hc}{e\lambda_2} - \frac{W_0}{e} \right] - \left[\frac{hc}{e\lambda_1} - \frac{W_0}{e} \right] = \frac{hc}{e} \left[\frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right]$$

$$\Delta V_0 = \frac{hc}{e} \left[\frac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2} \right]$$

$$\begin{aligned} \therefore \frac{hc}{e} &= \frac{\Delta V_0 \cdot \lambda_1 \lambda_2}{\lambda_1 - \lambda_2} \\ &= \frac{(1.3 - 0.9) \times 4000 \times 10^{-10} \times 4500 \times 10^{-10}}{500 \times 10^{-10}} \\ &= 1.44 \times 10^{-6} \text{ Vm} \end{aligned}$$

Also, $V_0 = \frac{hc}{e\lambda} - \frac{W_0}{e}$

$$\therefore \frac{W_0}{e} = \frac{hc}{e\lambda} - V_0 = \frac{1.44 \times 10^{-6}}{4000 \times 10^{-10}} - 1.3 = 2.3 \text{ V}$$

$$\Rightarrow W_0 = 2.3 \text{ eV}$$

ILLUSTRATION 5.19

A small piece of cesium metal ($\phi = 1.9 \text{ eV}$) is kept at a distance of 17.7 cm from a large metal plate having a charge density of $1.0 \times 10^{-9} \text{ C/m}^2$ on the surface facing the cesium piece. A monochromatic light of wavelength 400 nm is incident on the cesium piece. Find the minimum and maximum kinetic energy of the photoelectrons reaching the large metal plate. Neglect any change in electric field due to the small piece of cesium present.

Sol. Electric field between the plate, $E = \frac{\sigma}{\epsilon_0}$

The potential difference between the plates $V = E \cdot d = \frac{\sigma}{\epsilon_0} \cdot d$

$$\Rightarrow V = \frac{1.0 \times 10^{-9} \times 17.7}{8.85 \times 10^{-12} \times 100} = 20 \text{ V}$$

$$\text{From Einstein equation } K_{\max} = \frac{hc}{\lambda} - W_0 = \frac{12400}{4000} - 1.9 = 1.2 \text{ eV}$$

$$\text{But } K_{\max} = eV_0$$

$$\Rightarrow 1.2 \text{ eV} = eV_0$$

$$\Rightarrow V_0 = 1.2 \text{ V}$$

As V_0 is less than the potential difference between the plates (20 V). Hence the minimum energy required to reach the charged plate should be $K_{\min} = 20 \text{ eV}$.

For maximum kinetic energy, the potential difference between the plates should be an accelerating one. Hence maximum kinetic energy of electron should be

$$K_{\max} = 20 + 1.2 = 21.2 \text{ eV}$$

ILLUSTRATION 5.20

Calculate the velocity of a photoelectron if the work function of the target material is 1.24 eV and the wavelength of incident light is $4.36 \times 10^{-7} \text{ m}$. What retarding potential is necessary to stop the emission of the electrons?

Sol. We know that

$$\frac{1}{2}mv^2 = hf - \phi \text{ or } \frac{1}{2}mv^2 = \frac{hf}{\lambda} - \phi$$

Substituting the values, we get

$$v = 7.43 \times 10^5 \text{ m s}^{-1}$$

Let V_s be the retarding potential required to stop the emission of photoelectrons. Then, we have

$$eV_s = \frac{1}{2}mv^2$$

$$\Rightarrow V_s = \frac{\frac{1}{2}mv^2}{e} = \frac{2.511 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.58 \text{ V}$$

ILLUSTRATION 5.21

The stopping potential for photoelectrons emitted from a surface illuminated by light of wavelength 5893 Å is 0.36 V. Calculate the maximum kinetic energy of photoelectrons, the work function of the surface, and the threshold frequency.

Sol. We know that $K_{\max} = hf - \phi = \left(\frac{hc}{\lambda}\right) - \phi$

$$\text{or } \phi = \frac{hc}{\lambda} - K_{\max}$$

$$\text{Also, } K_{\max} = eV_s = 0.36 \text{ eV}$$

$$\Rightarrow \phi = \frac{(6.62 \times 10^{-34}) \times (3 \times 10^8)}{5893 \times 10^{-10}} - 0.36 \times 1.6 \times 10^{-19}$$

$$= 1.746 \text{ eV}$$

The threshold frequency is given by

$$f_0 = \frac{\phi}{h} = \frac{2.794 \times 10^{-19}}{6.62 \times 10^{-34}} = 4.22 \times 10^{14} \text{ Hz}$$

ILLUSTRATION 5.22

Photoelectric threshold of silver is $\lambda = 3800 \text{ Å}$. Ultraviolet light of $\lambda = 2600 \text{ Å}$ is incident on a silver surface. Calculate:

- the value of work function in joule and in eV.
 - maximum kinetic energy of the emitted photoelectrons.
 - the maximum velocity of the photoelectrons.
- (Mass of the electrons = $9.11 \times 10^{-31} \text{ kg}$)

Sol.

$$\text{(a) } \lambda_0 = 3800 \text{ Å}$$

$$W_0 = hf_0 = h \frac{c}{\lambda_0} = \frac{6.633 \times 10^{-34} \times 3 \times 10^8}{3800 \times 10^{-10}} \text{ J}$$

$$= 5.23 \times 10^{-19} \text{ J} = 3.27 \text{ eV}$$

$$\text{(b) Incident wavelength, } \lambda = 2600 \text{ Å}$$

$$\text{Therefore, } f = \text{incident frequency} = \frac{3 \times 10^8}{2600 \times 10^{-10}} \text{ Hz}$$

$$\text{Then, } K_{\max} = hf - W_0$$

$$hf = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2600 \times 10^{-10}} = 7.65 \times 10^{-19} \text{ J} = 4.78 \text{ eV}$$

$$K_{\max} = hf - W_0 = 4.78 \text{ eV} - 3.27 \text{ eV} = 1.51 \text{ eV}$$

$$\text{(c) } K_{\max} = \frac{1}{2}mv_{\max}^2$$

$$\Rightarrow v_{\max} = \sqrt{\frac{2K_{\max}}{m}} = \sqrt{\frac{2 \times 2.42 \times 10^{-19}}{9.11 \times 10^{-31}}}$$

$$= 0.7289 \times 10^6 \text{ m s}^{-1}$$

ILLUSTRATION 5.23

When a surface is irradiated with light of wavelength 4950 Å, a photocurrent appears which vanishes if a retarding potential greater than 0.6 V is applied across the photo tube. When a different source of light is used, it is found that the critical retarding potential is changed to 1.1 V. Find the work function of the emitting surface and the wavelength of the second source.

Sol. We know that $hf_1 = \phi + \frac{1}{2}mv_1^2$ and $\frac{1}{2}mv_1^2 = eV_1$

$$\begin{aligned}\phi &= hf_1 - eV_1 = \frac{hc}{\lambda_1} - eV_1 \\ &= \frac{(6.6 \times 10^{-34}) \times (3 \times 10^8)}{4950 \times 10^{-10}} - (1.6 \times 10^{-19})(0.6) \\ &= 3.04 \times 10^{-19} \text{ V} \\ hf_2 &= \phi + eV_2 \\ \frac{hc}{\lambda_2} &= 3.04 \times 10^{-19} + (1.6 \times 10^{-19}) \times 1.1 = 4.8 \times 10^{-19} \\ \Rightarrow \lambda_2 &= \frac{(6.6 \times 10^{-34}) \times (3 \times 10^8)}{4.8 \times 10^{-19}} = 4125 \text{ \AA}\end{aligned}$$

ILLUSTRATION 5.24

A beam of light has three wavelengths 4144 Å, 4972 Å, and 6216 Å with a total intensity of $3.6 \times 10^{-3} \text{ Wm}^{-2}$ equally distributed among the three wavelengths. The beams fall normally on an area 1.0 cm^2 of a clean metallic surface of work function 2.3 eV. Assume that there is no loss of light by reflection and that each energetically capable photon ejects one electron. Calculate the number of photoelectrons liberated in 2 s.

Sol. We know that threshold wavelength $(\lambda_0) = \frac{hc}{\phi}$.

$$\lambda_0 = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{2.3 \times (1.6 \times 10^{-19})} = 5.404 \times 10^{-7} \text{ m} = 5404 \text{ \AA}$$

Thus, wavelengths 4144 Å and 4972 Å will emit electrons from the metal surface.

Energy incident on the surface per unit time for each wavelength
= Intensity of each wavelength $\times$ Area of the surface

$$= \frac{3.6 \times 10^{-3}}{3} \times (1.0 \text{ cm}^2) = 1.2 \times 10^{-7} \text{ W}$$

Energy incident on the surface for each wavelength in 2 s,

$$E = (1.2 \times 10^{-7}) \times (2) = 2.4 \times 10^{-7} \text{ J}$$

Number of photons n_1 due to wavelength 4144 Å
(= $4144 \times 10^{-10} \text{ m}$),

$$n_1 = \frac{(2.4 \times 10^{-7})(4144 \times 10^{-10})}{(6.63 \times 10^{-34})(3 \times 10^8)} = 0.5 \times 10^{12}$$

Number of photons n_2 due to wavelength 4972 Å,

$$n_2 = \frac{(2.4 \times 10^{-7})(4972 \times 10^{-10})}{(6.63 \times 10^{-34})(3 \times 10^8)} = 0.575 \times 10^{12}$$

$$N = n_1 + n_2 = 0.5 \times 10^{12} + 0.575 \times 10^{12} = 1.075 \times 10^{12}$$

ILLUSTRATION 5.25

A photocell is operating in saturation mode with a photocurrent $4.8 \mu\text{A}$ when a monochromatic radiation of wavelength 3000 Å and power 1 mW is incident. When another monochromatic radiation of wavelength 1650 Å and power 5 mW is incident, it is observed that maximum velocity of photoelectron increases to two times. Assuming efficiency of photoelectron generation per incident to be same for both the cases, calculate:

- threshold wavelength for the cell
- efficiency of photoelectron generation
[(Number of photoelectrons emitted per incident photon) $\times$ 100]
- saturation current in the second case.

Sol.

$$(a) K_1 = \frac{12400}{3000} - W = 4.13 - W \quad \dots(i)$$

$$K_2 = \frac{12400}{1650} - W = 7.51 - W \quad \dots(ii)$$

$$\text{Since } V_2 = 2V_1, \text{ so } K_2 = 4K_1 \quad \dots(iii)$$

Solving above equations, we get $W = 3 \text{ eV}$

$$\text{Therefore, threshold wavelength, } \lambda_0 = \frac{12400}{3} = 4133 \text{ \AA}$$

$$(b) \text{ Energy of a photon in the first case} = \frac{12400}{3000} = 4.13 \text{ eV}$$

$$\text{or } E_1 = 6.6 \times 10^{-19} \text{ J}$$

Rate of incident photons (number of photons per second)

$$\frac{P_1}{E_1} = \frac{10^{-3}}{6.6 \times 10^{-19}} = 1.52 \times 10^{15} \text{ s}^{-1}$$

$$\text{Number of electrons ejected} = \frac{4.8 \times 10^{-6}}{1.6 \times 10^{-19}} \text{ s}^{-1} = 3.0 \times 10^{13} \text{ s}^{-1}$$

Therefore, efficiency of photoelectron generation,

$$\gamma = \frac{3.0 \times 10^{13}}{1.52 \times 10^{15}} \times 100 = 1.97\%$$

(c) Energy of photon in the second case,

$$E_2 = \frac{12400}{1650} = 7.51 \text{ eV} = 12 \times 10^{-19} \text{ J}$$

Therefore, number of photons incident per second,

$$n_2 = \frac{P_2}{E_2} = \frac{5.0 \times 10^{-3}}{12 \times 10^{-19}} = 4.17 \times 10^{15} \text{ s}^{-1}$$

Number of electrons emitted per second is

$$\frac{1.97}{100} \times 4.17 \times 10^{15} = 9.27 \times 10^{13} \text{ s}^{-1}$$

Therefore, saturation current in the second case

$$i = (9.27 \times 10^{13}) (1.6 \times 10^{-19}) \text{ A} = 14.8 \mu\text{A}$$

CONCEPT APPLICATION EXERCISE 5.2

- Lithium has a work function of 2.30 eV. It is exposed to light of wavelength 4800 Å. Find the maximum kinetic energy with which electron leaves the surface. What is the longest wavelength which can produce the photoelectrons?
- Electrons with maximum kinetic energy 3 eV are ejected from a metal surface by ultraviolet radiation of wavelength 1500 Å. Determine the work function of the metal, the threshold wavelength of metal and the stopping potential difference required to stop the emission of electrons.
- Light of wavelength 400 nm is incident continuously on a Cesium ball, (work function 1.9 eV). Find the maximum potential to which the ball will be charged
- One milliwatt of light of wavelength 4560 Å is incident on a cesium surface. Calculate the photoelectric current liberated

assuming a quantum efficiency of 0.5%, given Planck's constant $h = 6.62 \times 10^{-34}$ J-s and velocity of light $c = 3 \times 10^8$ m/s.

5. In a photoelectric experiment, the collector plate is at 2.0 V with respect to the emitter plate made of copper ($W_0 = 4.5$ eV). The emitter is illuminated by a source of monochromatic light of wavelength 200 nm. Find the minimum and maximum kinetic energy of the photoelectrons reaching the collector.
6. Radiations of frequencies f_1 and f_2 are made to fall in turn, on a photo-sensitive surface. The stopping potentials required for stopping the most energetic emitted photoelectrons in the two cases are V_1 and V_2 respectively. Obtain a formula for determining Planck's constant and the threshold frequency in terms of these parameters.
7. If the frequency of incident light on a metal surface is doubled, will the kinetic energy of the photoelectrons be doubled?
8. A light of wavelength 1240 Å falls on a metallic sphere of radius 1 m and work function $W_0 = 3$ eV. Find the maximum number of electrons left from the sphere till the photoelectric effect stops.
9. All electrons ejected from a surface by incident light of wavelength 2000 Å can be stopped before travelling 1 m in the direction of uniform electric field of 4 N/C. The work function of the surface is

ANSWERS

1. 0.28 eV, 5391.3 Å 2. 5.26 eV, 2357.4 Å, 3.0 V
3. 1.2 V 4. 1.824 μA 5. 2 eV, 3.7 eV
8. 4.68×10^9 9. 2.2 eV

WAVE NATURE OF MATTER: DE-BROGLIE WAVES OR MATTER WAVES

The experiments related to interference, diffraction and polarization confirm the wave nature of light, whereas the photoelectric effect and Compton effect put the corpuscular or particle nature of light. It means light act as wave and particle both. This ideal of dual behaviour of the photon inspired Louis de-Broglie, to put forth a revolutionary hypothesis. He suggested that nature loves symmetry. If a light wave can behave like a particle on some occasions, it is reasonable to suppose that a particle also has a dual nature. In some circumstances, a particle may behave like a wave rather than a particle.

DE-BROGLIE WAVES EQUATION

The two physical quantities which govern all the forms of the physical universe are mass and energy. These quantities are connected through the Einstein's mass-energy relationship:

$$E = mc^2 \quad \dots(i)$$

where E = energy of a photon

m = relativistic mass of a photon

and c = velocity of light in vacuum.

Above equation shows that there is a complete equivalence between matter (mass) and radiation (energy). There must be a mutual symmetry between matter (mass) and radiation.

According to the quantum theory of radiation,

$$E = hf \quad \dots(ii)$$

where h = Planck's constant

f = frequency of the photon

From Eq. (i), relativistic mass of a photon of energy (E) is given by $m = \frac{E}{c^2}$

If a photon moves with the velocity of light, momentum of the photon, i.e.,

$$p = mc = \frac{E}{c^2} \times c = \frac{E}{c} \quad \dots(iii)$$

From Eqs. (ii) and (iii), $p = \frac{hf}{c} \quad \dots(iv)$

$$\text{As } c = f\lambda, \frac{f}{c} = \frac{1}{\lambda}$$

where λ is the wavelength of the photon.

$$\text{From Eq. (iv), } p = \frac{h}{\lambda} \quad \dots(v)$$

A quantum of light, i.e., photon should thus be able to act like particle and possess momentum.

Now, if there is an electron (or any other particle) of mass m moving with velocity v , its momentum is given by

$$p = mv \quad \dots(vi)$$

From Eqs. (v) and (vi), we get $\frac{h}{\lambda} = mv$

$$\text{or } \lambda = \frac{h}{mv} = \frac{h}{p} \quad \dots(vii)$$

This is known as de-Broglie's wave equation.

$$\text{Also, } K = \frac{p^2}{2m} \Rightarrow \lambda = \frac{h}{\sqrt{2mK}}$$

Thus, from here we conclude that a particle of mass m moving with a velocity v , can be considered to be a wave of wavelength λ . This equation explains the dual nature of matter as it connects the wave characteristic ' λ ' with the particle characteristic ' p '.

From de-Broglie's equation, we find that

1. The wavelength of a moving particle is inversely proportional to its momentum.

$$\text{i.e., } \lambda \propto \frac{1}{p}$$

2. If $v = 0$, then $\lambda = \infty$. This implies that waves are associated with material particles only when they are in motion.
3. To be associated with a de-Broglie wave, a particle need not have a charge. That is why, de-Broglie waves are also known as matter waves.
4. de-Broglie waves cannot be electromagnetic in nature because electromagnetic waves are only associated with accelerated charged particles.

Important Point:

The de-Broglie wavelength is independent of the charge and nature of the material particle.

de-Broglie wavelength, $\lambda \propto 1/m$. This wavelength is significantly measurable (of the order of atomic-planes spacing in crystals) only in case of subatomic particles like electrons, protons, etc.; due to the smallness of their masses. But the de-Broglie wavelength of large moving objects is very small, quite beyond measurement, due to their large masses. That is why the **macroscopic objects in our daily life do not show wave-like properties.**

ILLUSTRATION 5.26

Calculate the de-Broglie wavelengths for electrons and protons, if their speed is 10^{15} m/s.

Sol. We are given that $v = 10^5$ m/s.

(i) Mass of an electron, $m = 9.1 \times 10^{-31}$ kg

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{(9.1 \times 10^{-31})10^5}$$

$$\text{or } \lambda = 72.8 \times 10^{-10} \text{ m} = 72.8 \text{ \AA}$$

(ii) Mass of proton, $m = 1.67 \times 10^{-27}$ kg

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{(1.67 \times 10^{-27})10^5}$$

$$\text{or } \lambda = 0.03968 \times 10^{-10} \text{ m} = 0.03968 \text{ \AA}$$

ILLUSTRATION 5.27

An electron and a photon have the same de-Broglie wavelength. Which one of these has higher kinetic energy?

Sol. Let λ be the de-Broglie wavelength of the electron and the photon. If m and v are the mass and the velocity of the electron, then de-Broglie wavelength of the electron

$$\lambda = \frac{h}{mv}$$

The photon has zero rest mass. Therefore, energy of the photon is totally kinetic in nature. Since the wavelength of the photon is same as that of the electron, the kinetic energy of the photon having wavelength λ ,

$$E_1 = \frac{hc}{\lambda} = \frac{hc}{h/mv}$$

$$\text{or } E_1 = mvc \quad \dots(i)$$

Now, the kinetic energy of the electron,

$$E_2 = \frac{1}{2}mv^2 = mv \times \frac{v}{2} \quad \dots(ii)$$

Since $c > v/2$ (as $c > v$), from the results (i) and (ii), it follows that

$$E_1 > E_2$$

i.e., kinetic energy of the photon is greater than that of the electron. As it moves with the speed c , it is faster than electron.

ILLUSTRATION 5.28

A photon and an electron have the same de-Broglie wavelength. Which has greater total energy? Explain.

Sol. Let λ be the de-Broglie wavelength of the electron and also the wavelength of the photon. If m and v are the mass and the velocity of the electron, then de-Broglie wavelength of the

$$\text{electron } \lambda = \frac{h}{mv}$$

$$\text{Energy of the photon having wavelength } \lambda, E_1 = \frac{hc}{\lambda}$$

Since wavelength of the photon is same as that of the electron,

$$E_1 = hc \times \frac{mv}{h}$$

$$\text{or } E_1 = mvc \quad \dots(i)$$

According to Einstein's mass-energy equivalence relation, the energy of the electron (mass m)

$$E_2 = mc^2 = mc \times (c) \quad \dots(ii)$$

Since $c > v$, from Eqs. (i) and (ii), it follows that

$$E_2 > E_1$$

i.e., the total energy of electron is greater than that of the photon.

DE-BROGLIE WAVELENGTH OF AN ELECTRON

Consider an electron of mass m and charge e . Let v be the final velocity attained by the electron when it is accelerated from rest through a potential difference of V volts. Then kinetic energy gained by the electron equals the work done on the electron by the electric field.

$$\text{KE gained by the electron, } K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

Work done on the electron = eV

$$\therefore K = \frac{p^2}{2m} = eV$$

$$\text{or } p = \sqrt{2mK} = \sqrt{2meV}$$

Hence the de-Broglie wavelength of the electron is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2meV}}$$

$$\text{Now } h = 6.63 \times 10^{-34} \text{ Js}$$

$$m = 9.1 \times 10^{-31} \text{ kg}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

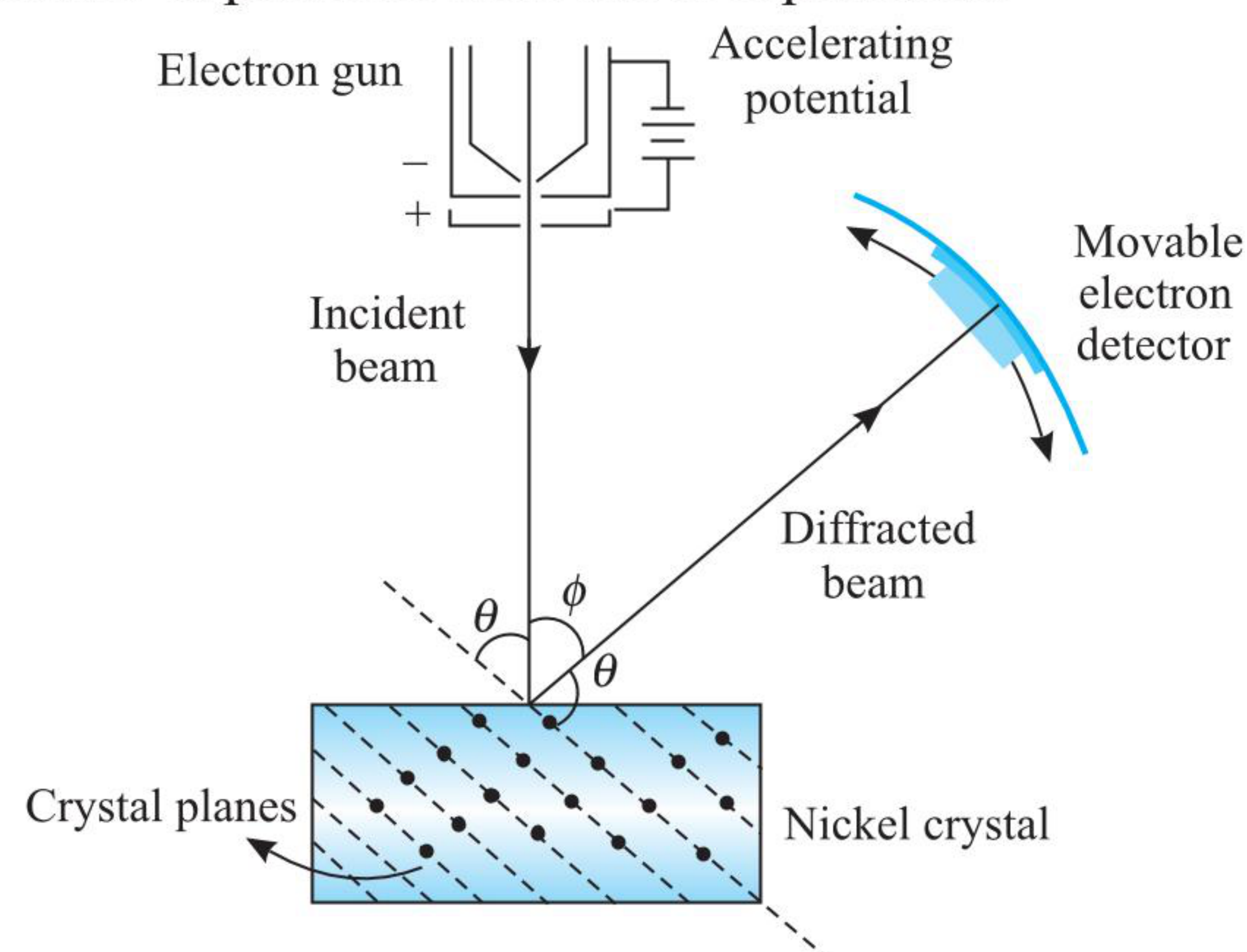
$$\therefore \lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} V}}$$

$$\Rightarrow \lambda = \frac{12.27 \times 10^{-10}}{\sqrt{V}} \text{ m} = \frac{12.27}{\sqrt{V}} \text{ \AA}$$

For an accelerating potential of 120 V, we find $\lambda = 0.112$ nm. This wavelength is of the same order as the spacing between the atomic planes in crystals. This suggested that matter waves associated with electrons could be detected by electron diffraction experiments.

EXPERIMENTAL DEMONSTRATION OF WAVE NATURE OF ELECTRONS**Davisson and Germer Experiment**

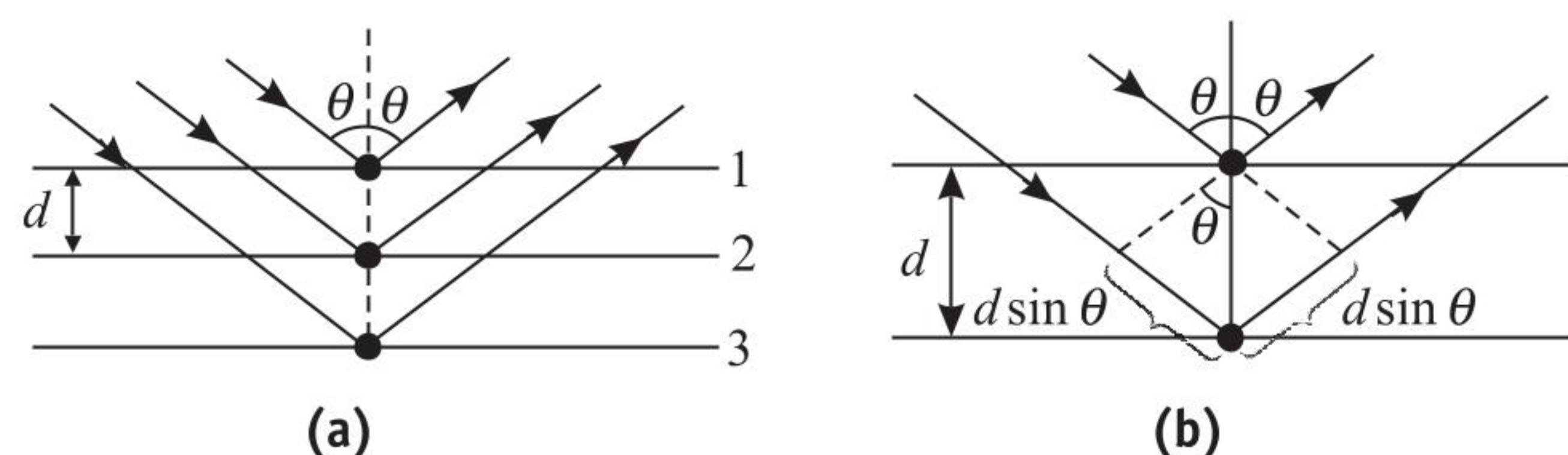
Davisson and Germer confirmed the de-Broglie theory about the wave nature of particles with direct experiment.



Experimental arrangement: The apparatus used by Davisson and Germer is shown in figure. Here the electrons emitted by the hot filament of an electron gun are accelerated by applying a suitable potential difference. These electrons are directed at the **nickel crystal**. The crystal is capable of rotation about an axis perpendicular to the plane of paper. The electrons, scattered in different directions by the atoms of Ni crystal, are received by a movable detector which is just an electron collector. Thus, we measure scattered electron intensity as a function of the scattering angle ϕ , the angle between the incidence and the scattered electron beam. The experiment is repeated for different accelerating potentials V .

Observations and inference: Above result satisfies the Bragg's law of diffraction of X-rays.

According to Bragg's, $2d \sin \theta = n\lambda$



where θ = diffraction angle
 d = lattice constant, distance between crystal planes
 n = order of maxima

The value of d , as determined by X-ray diffraction by Ni-crystal, is $0.91 \text{ \AA}$. As $\theta = 65^\circ$ for first order diffraction maxima, so $2d \sin \theta = n\lambda$

$$\text{or } 2 \times 0.91 \text{ \AA} \sin 65^\circ = 1\lambda$$

$$\Rightarrow \lambda = 1.65 \text{ \AA}$$

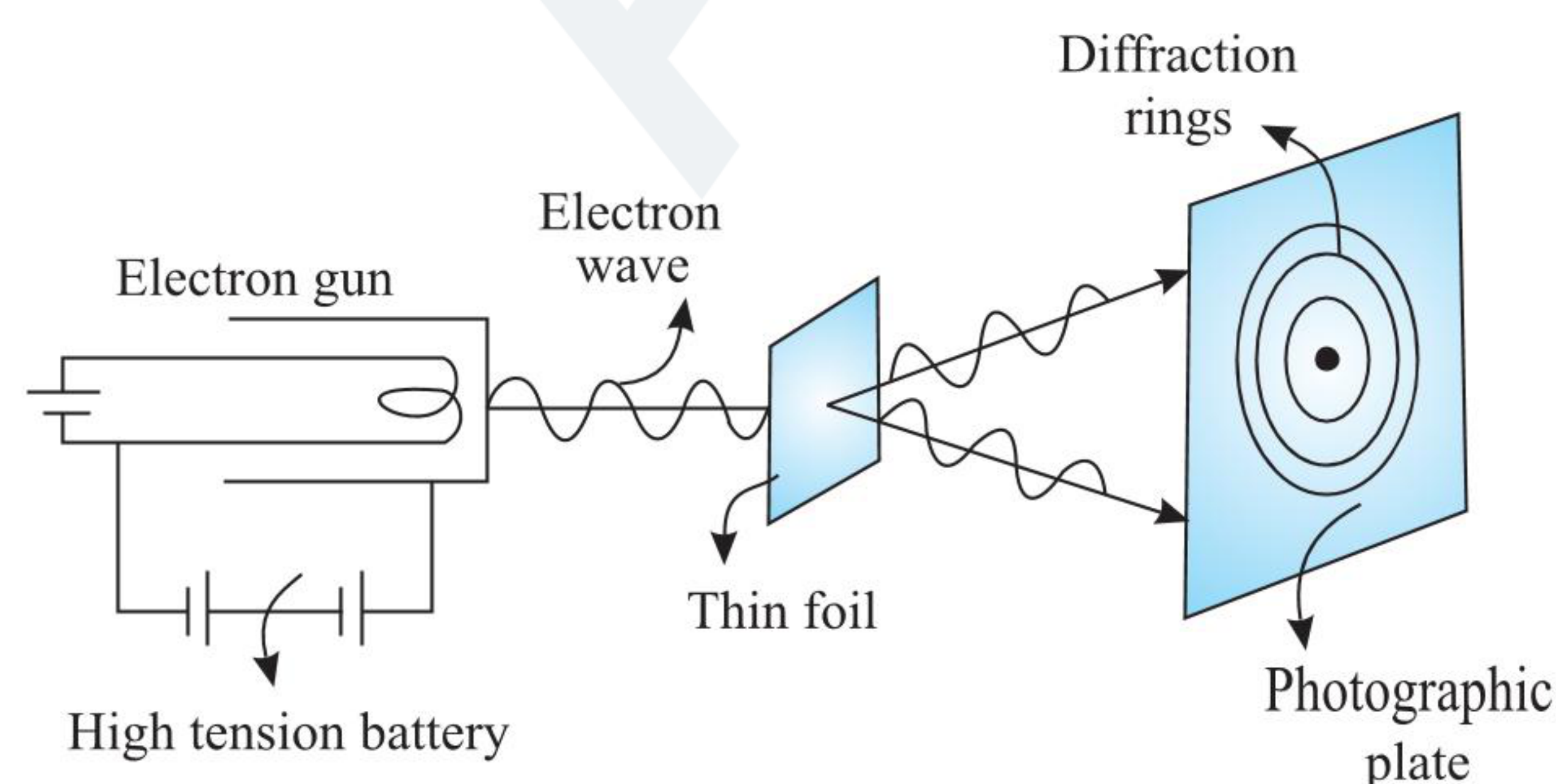
Now according to de-Broglie hypothesis, the wavelength for electrons accelerated through a potential difference of 54 volt is given by

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ \AA} = \frac{12.27}{\sqrt{54}} = 1.67 \text{ \AA}$$

Thus, the two results obtained are in remarkable agreement with each other. We conclude that particles, like waves, can be diffracted. Thus, Davisson–Germer experiment provides a direct verification of the de-Broglie hypothesis regarding the wave nature of particles.

G.P. Thomson's Experiment

G.P. Thomson was able to obtain a diffraction pattern of an electron beam. The experimental set-up is shown in figure. From an electron gun, a beam of electrons accelerated through a potential difference of 10 to 50 kV is incident on a thin platinum foil.



The emergent beam is received on a photographic plate. The electron beam is diffracted at the spacings between the randomly oriented crystals of the thin foil. On the photographic plate, we get a circular diffraction pattern similar to Laue's X-ray diffraction pattern. This conclusively proves that **the electron beams behave like waves**.

DE-BROGLIE WAVELENGTH ASSOCIATED WITH DIFFERENT CHARGED PARTICLES

The de Broglie wavelength of a charged particle is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}}$$

Here, $h = 6.63 \times 10^{-34} \text{ Js}$

Particle	Mass	Charge	Wavelength
Protons	$1.67 \times 10^{-27} \text{ kg}$	$1.6 \times 10^{-19} \text{ C}$	$\lambda = \frac{0.286}{\sqrt{V}} \text{ \AA}$
Deuterons	$2 \times 1.67 \times 10^{-27} \text{ kg}$	$1.6 \times 10^{-19} \text{ C}$	$\lambda = \frac{0.202}{\sqrt{V}} \text{ \AA}$
α -particles	$4 \times 1.67 \times 10^{-27} \text{ kg}$	$2 \times 1.6 \times 10^{-19} \text{ C}$	$\lambda = \frac{0.101}{\sqrt{V}} \text{ \AA}$

DE-BROGLIE WAVELENGTH ASSOCIATED WITH UNCHARGED PARTICLES

1. For neutrons ($m_n = 1.67 \times 10^{-27} \text{ kg}$):

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} E}}$$

2. For thermal neutrons at ordinary temperatures:

$$E = kT$$

$$\lambda = \frac{h}{\sqrt{2mkT}} = \frac{30.835}{\sqrt{T}} \text{ \AA}$$

3. For gas molecules: $\lambda = \frac{h}{m \times C_{\text{rms}}}$

4. For gas molecules at $T \text{ K}$: $E = \frac{3}{2}kT \Rightarrow \lambda = \frac{h}{\sqrt{3mkT}}$

ILLUSTRATION 5.29

Calculate the de-Broglie wavelength associated with an α -particle accelerated through a potential difference of 200 V. Given $m_p = 1.67 \times 10^{-27} \text{ kg}$.

Sol. Mass of α -particle, $m = 4m_p = 4 \times 1.67 \times 10^{-27} \text{ kg}$

Charge on α -particle = $2e$

If the α -particle acquires velocity v , then $qV = \frac{1}{2}mv^2$

$$\text{or } 2eV = \frac{1}{2}mv^2$$

$$\therefore mv = \sqrt{4meV}$$

$$\Rightarrow \lambda = \frac{h}{mv} = \frac{h}{\sqrt{4meV}}$$

$$\Rightarrow \lambda = \frac{6.6 \times 10^{-34}}{\sqrt{4 \times 4 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-19} \times 200}} \text{ m}$$

$$= 0.07138 \times 10^{-11} \text{ m} = 0.007138 \text{ \AA}$$

ILLUSTRATION 5.30

Find the typical de-Broglie wavelength of an electron in a metal at 27°C and compare it with the mean separation between two electrons in a metal which is given to be about $2 \times 10^{-10} \text{ m}$.

Sol. Mass of an electron is $m = 9.11 \times 10^{-31} \text{ kg}$

and $T = 27 + 273 = 300 \text{ K}$

$\therefore$ de-Broglie wavelength of electron is

$$\lambda = \frac{h}{\sqrt{3mkT}} = \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 9.11 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}} \text{ m}$$

$$\Rightarrow \lambda = 6.2 \times 10^{-9} \text{ m}$$

Mean separation between two electrons in a metal is

$$r = 2 \times 10^{-10} \text{ m}$$

$$\therefore \frac{\lambda}{r} = \frac{6.2 \times 10^{-9}}{2 \times 10^{-10}} = 31$$

Thus the de-Broglie wavelength is much greater than the given interelectron separation.

ILLUSTRATION 5.31

The wavelength of light from the spectral emission line of sodium is 589 nm . Find the kinetic energy at which (a) an electron and (b) a neutron would have the same de Broglie wavelength. Given that mass of neutron $= 1.66 \times 10^{-27} \text{ kg}$.

Sol. Here, $\lambda = 589 \text{ nm} = 589 \times 10^{-9} \text{ m}$

The de Broglie wavelength of a particle having energy E is given by

$$\lambda = \frac{h}{\sqrt{2mE}} \quad \text{or} \quad E = \frac{h^2}{2m\lambda^2}$$

(a) For electron: Here, $\lambda = 589 \text{ nm} = 589 \times 10^{-9} \text{ m}$ and

$$m = 9.1 \times 10^{-31} \text{ kg}$$

$$\therefore E = \frac{h^2}{2m\lambda^2} = \frac{(6.62 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times (589 \times 10^{-9})^2}$$

$$= 6.94 \times 10^{-25} \text{ J}$$

(b) For neutron: Here $\lambda = 589 \text{ nm} = 589 \times 10^{-9} \text{ m}$ and

$$m = 1.66 \times 10^{-27} \text{ kg}$$

$$\therefore E = \frac{h^2}{2m\lambda^2} = \frac{(6.62 \times 10^{-34})^2}{2 \times 1.66 \times 10^{-27} \times (589 \times 10^{-9})^2}$$

$$= 3.81 \times 10^{-28} \text{ J}$$

ILLUSTRATION 5.32

(a) For what kinetic energy of a neutron will the associated de-Broglie wavelength be $1.40 \times 10^{-10} \text{ m}$?

(b) Also, find the de-Broglie wavelength of a neutron, in thermal equilibrium with matter, having an average kinetic energy of $(3/2) kT$ at 300 K . Given that mass of neutron $= 1.66 \times 10^{-27} \text{ kg}$ and $k = 1.38 \times 10^{-23} \text{ J kg}^{-1}$.

Sol.

(a) Here $\lambda = 1.40 \times 10^{-10} \text{ m}$; $m = 1.66 \times 10^{-27} \text{ kg}$

The energy of neutron is given by $E = \frac{h^2}{2m\lambda^2}$

$$= \frac{(6.62 \times 10^{-34})^2}{2 \times 1.66 \times 10^{-27} \times (1.40 \times 10^{-10})^2} = 6.735 \times 10^{-21} \text{ J}$$

(b) Here $k = 1.38 \times 10^{-23} \text{ J kg}^{-1}$ and $T = 300 \text{ K}$

Therefore, energy of neutron

$$E = \frac{3}{2} kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300 = 6.21 \times 10^{-21} \text{ J}$$

$$\text{Now, } \lambda = \frac{h}{\sqrt{2mE}} = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 1.66 \times 10^{-27} \times 6.21 \times 10^{-21}}}$$

$$= 1.46 \times 10^{-10} \text{ m} = 1.46 \text{ \AA}$$

ILLUSTRATION 5.33

Determine the de-Broglie wavelength of a proton, whose kinetic energy is equal to the rest of the mass energy of an electron. Given that the mass of an electron is $9.1 \times 10^{-31} \text{ kg}$ and the mass of a proton is 1837 times as that of the electron.

Sol. Here mass of electron, $m_0 = 9.1 \times 10^{-31} \text{ kg}$

Mass of proton, $m = 1837 \times 9.1 \times 10^{-31} = 1.67 \times 10^{-27} \text{ kg}$

Let v be the velocity of the proton. Then, $\frac{1}{2} mv^2 = m_0 c^2$

$$\text{or } m^2 v^2 = 2mm_0 c^2$$

$$\text{or } mv = \sqrt{2mm_0} c$$

$$= \sqrt{2 \times 1.67 \times 10^{-27} \times 9.1 \times 10^{-31}} \times 3 \times 10^8$$

$$= 1.654 \times 10^{-20} \text{ kg m s}^{-1}$$

Therefore, de-Broglie wavelength of the proton,

$$\lambda = \frac{h}{mv} = \frac{6.62 \times 10^{-34}}{1.654 \times 10^{-20}} = 4 \times 10^{-14} \text{ m}$$

ILLUSTRATION 5.34

Find the ratio of de Broglie wavelength of molecules of hydrogen and helium which are at temperatures 27°C and 127°C , respectively.

Sol. de-Broglie wavelength is given by $\lambda = h/mv$.

Root mean square velocity of a gas particle at the given temperature (T) is given as, $\frac{1}{2} mv^2 = \frac{3}{2} kT$

$$v = \sqrt{\frac{3kT}{m}}$$

where k = Boltzmann's constant, m = mass of the gas particle, and T = temperature of the gas in K.

$$mv = \sqrt{3mkT}$$

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{3mkT}}$$

$$\frac{\lambda_H}{\lambda_{He}} = \sqrt{\frac{m_{He}T_{He}}{m_H T_H}} = \sqrt{\frac{(4)(273 + 127)}{(2)(273 + 27)}} = \sqrt{\frac{8}{3}}$$

ILLUSTRATION 5.35

The extent of localization of a particle is determined roughly by its de Broglie wave. If an electron is localized within the nucleus (of size about 10^{-14} m) of an atom, what is its energy? Compare this energy with the typical binding energies (of the order of a few MeV) in a nucleus and hence argue why electrons cannot reside in a nucleus.

Sol. The de Broglie wavelength of electron,

$$\lambda = \text{size of atom} = 10^{-14} \text{ m}$$

Corresponding to this wavelength, the electron possesses energy,

$$E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{10^{-14}} = 1.986 \times 10^{-11} \text{ J}$$

$$= \frac{1.986 \times 10^{-11}}{1.6 \times 10^{-13}} = 124.1 \text{ MeV}$$

If the electrons possess energy of the order of 124 MeV, they will not be able to stay inside the nucleus. It is because Coulomb's force cannot provide such high value of binding energy to the electrons. Hence, electrons cannot reside inside the nucleus.

PROPERTIES OF MATTER WAVES

1. Matter waves are different from electromagnetic waves. This is an obvious fact as matter waves are related to moving particles and independent of the fact whether particle is neutral or charged.
2. Matter waves travel with speed more than that of light.

$$v_{\text{matter waves}} = f\lambda = \frac{E}{h} \times \frac{h}{p} = \frac{mc^2}{mv} = \frac{c^2}{v}$$

As velocity of particle can never exceed speed of light, hence $v_{\text{matter waves}} > c$. As a matter of fact, matter waves travel with the body in a group. Group velocity is same as velocity of body, whereas the individual waves constituting the group may theoretically move with speed greater than that of light.

3. In ordinary situation, de-Broglie wavelength is very small and wave nature of matter can be ignored. To appreciate this point, let us calculate de-Broglie wavelength of 46 g of golf ball moving with a speed of 30 m s^{-1} .

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{(0.046)(30)} = 4.8 \times 10^{-34} \text{ m}$$

The wavelength of golf ball is so small compared with its dimensions that we would not expect to detect any wave aspects in its behavior using ordinary instrument.

4. The wave and particle aspects of moving bodies can never be observed at the same time, i.e., the two nature are mutually exclusive. Which nature dominates in a given situation will be directed by how its de Broglie wavelength compares with its dimensions and dimensions of whatever it interacts with.
5. The square of the amplitude of de Broglie wave at any point is proportional to the probability of finding the particle at that point.

ELECTRON MICROSCOPE

Electron microscope is an important application of de-Broglie waves designed to study very minute objects like viruses, microbes and the crystal structure of the solids.

Principle: Its working is based on the following facts:

1. Like light radiations, electron beams behave as waves but with much smaller wavelengths.
2. By using electric and magnetic fields, electron beams can be focused just as ordinary light beams are focused by glass lenses.

Theory: The de-Broglie wavelength associated with an electron accelerated through a potential difference of V volts is given by

$$\lambda = \frac{12.3}{\sqrt{V}} \text{ \AA}$$

The magnifying power of a microscope is inversely related with the wavelength of radiation used. With an optical microscope, a magnification of ≈ 1500 is possible. This limit is due to the large wavelength of visible radiation. However, by selecting a suitable value of potential difference V , one can have an electron beam of as small wavelength as desired. That is why an electron microscope can have a very high magnification of $\approx 100,000$. In an electron microscope, the electrons are focused with the help of electric and magnetic lenses.

ILLUSTRATION 5.36

The speed of electrons in an electron microscope is 10^8 m s^{-1} . If protons with the same speed are used instead of electrons, what additional advantage such a proton microscope has over an electron microscope?

Sol. de-Broglie wavelength, $\lambda = \frac{h}{mv}$

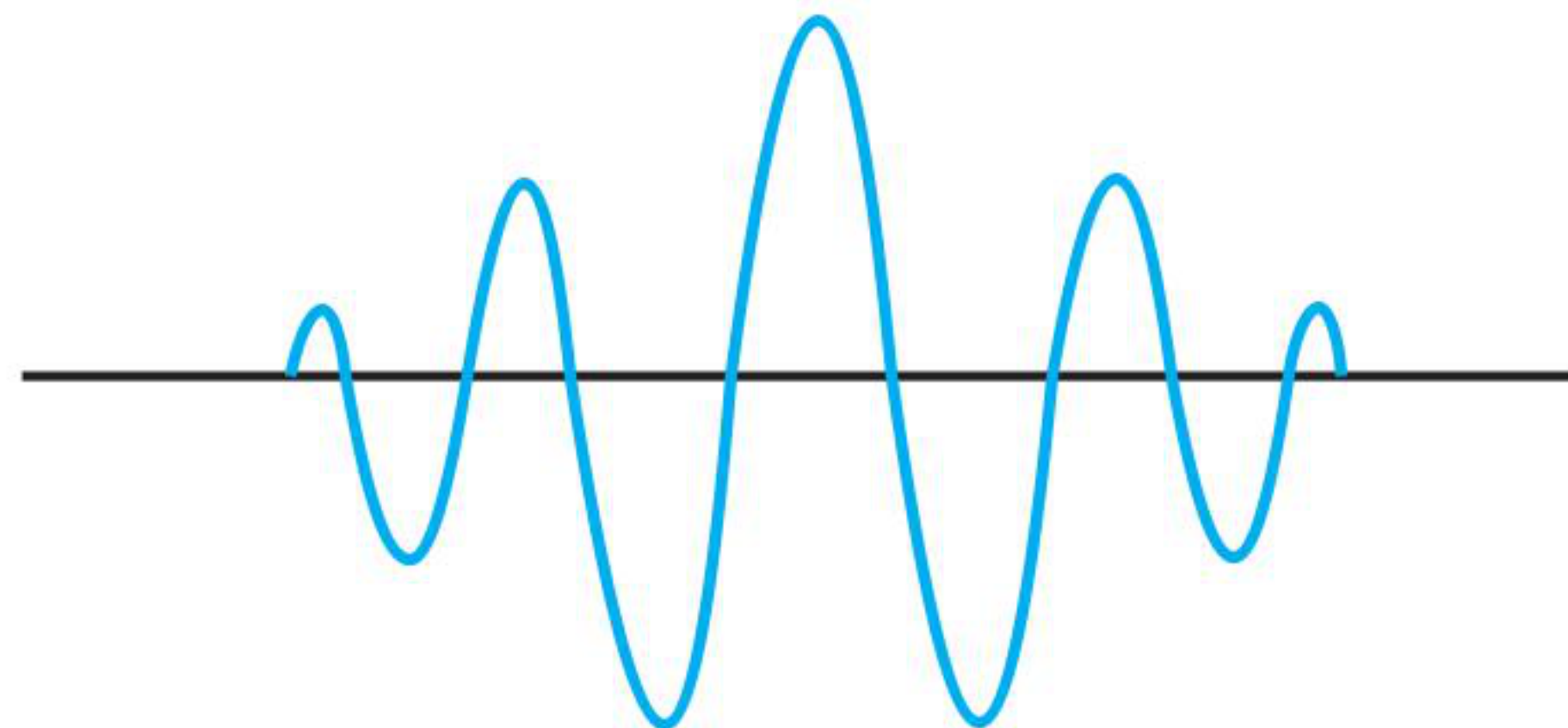
As the mass of a proton is 1836 times that of an electron, so for the same speed, the wavelength of proton beam will be $\frac{1}{1836}$ times that of an electron beam.

But resolving power of a microscope $\propto \frac{1}{\text{wavelength}}$.

Hence the resolving power of a proton microscope will be 1836 times that of an electron microscope.

MATTER WAVES AND HEISENBERG'S UNCERTAINTY PRINCIPLE

The de-Broglie wave associated with a moving particle should be regarded as a wave packet instead of a continuous waves (see figure). A wave packet is a localized wave consisting of continuous range of frequencies



The wave packet description of an electron

The matter wave picture given by de Broglie elegantly incorporates the **Heisenberg's uncertainty principle**. According to this principle, it is not possible to measure both the position and momentum of a subatomic particle at the same time exactly. The product of the uncertainty in position (Δx) and the uncertainty in momentum (Δp) is never less than $h/4\pi$,

$$\text{i.e., } \Delta x \times \Delta p \geq \frac{h}{4\pi}$$

It must be noted that the uncertainties in x and p are not due to imperfections in the apparatus. In fact, they are due to the fact that the quantities involved (x and p) are not precise.

Important Points:

- Heisenberg's uncertainty principle provides complementary descriptions of the same phenomenon. That is, to obtain the complete picture of any event, we need both the wave and the particle properties of matter, but because of the uncertainty principle, it is impossible to design an experiment that will show both of them in detail at the same time. Any one experiment will reveal the details of either the wave or the particle character, according to the purpose for which the experiment is designed.
- When the momentum of the particle is accurately known, $\Delta p = 0$ and $\Delta x \approx \infty$. Such a particle is represented by a wave of constant amplitude extending over all space as shown in figure.

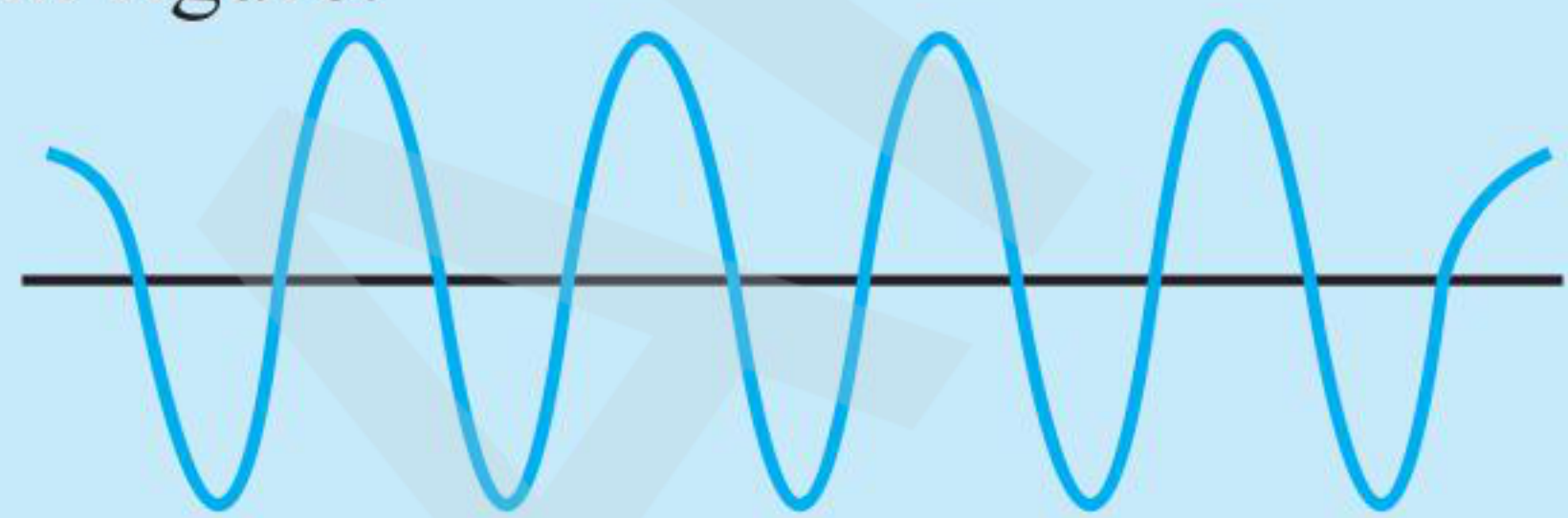


ILLUSTRATION 5.37

Applying uncertainty principle, calculate the minimum, kinetic energy of an electron confined in a region of length $l = 0.2 \text{ nm}$.

Sol. The uncertainty in position of the electron is $\Delta x = l$

Then, the uncertainty in momentum of the electron is $p \geq \frac{h}{4\pi\Delta x}$

$$\text{or } p \geq \frac{h}{4\pi l}$$

Hence, the minimum momentum is $p_m = \frac{h}{4\pi l}$

Then, the minimum KE of the electron is $K_{\min} = \frac{p_m^2}{2m}$

$$\begin{aligned} \text{or } K_{\min} &= \frac{h^2}{32\pi^2 ml^2} = \left[\frac{(6.3 \times 10^{-34})^2}{32 \times (3.14)^2 (9.1 \times 10^{-31}) (0.2 \times 10^{-9})^2} \right] \text{ J} \\ &\simeq 2.15 \times 10^{-5} \text{ eV} \quad (\because 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}) \end{aligned}$$

CONCEPT APPLICATION EXERCISE 5.3

1. A proton and an alpha particle are accelerated through the same potential. Which one of the two has (i) greater value of de-Broglie wavelength associated with it, and (ii) less kinetic energy? Justify your answer.
2. An electron and a proton are accelerated through the same potential. Which one of the two has (i) greater value of de-Broglie wavelength associated with it and (ii) less momentum? Justify your answer.
3. An electron, α -particle and a proton have the same kinetic energy. Which of these particles has the shortest de-Broglie wavelength?
4. An α -particle and a proton are accelerated from rest through the same potential difference V . Find the ratio of de-Broglie wavelengths associated with them.
5. Calculate the ratio of the accelerating potential required to accelerate a proton and an α -particle to have the same de-Broglie wavelength associated with them.
[Given: Mass of proton = $1.6 \times 10^{-27} \text{ kg}$; Mass of α -particle = $6.4 \times 10^{-27} \text{ kg}$]
6. An electron of mass m and charge e initially at rest gets accelerated by a constant electric field E . Find the rate of change of de-Broglie wavelength of this electron at time t .
7. Find the de Broglie's wavelength of a hydrogen molecule 27°C at its most probable speed.

ANSWERS

4. $1:2\sqrt{2}$ 5. 8:1 6. $\frac{-h}{eEt^2}$ 7. $1.2 \times 10^{-7} \text{ m}$

Exercises

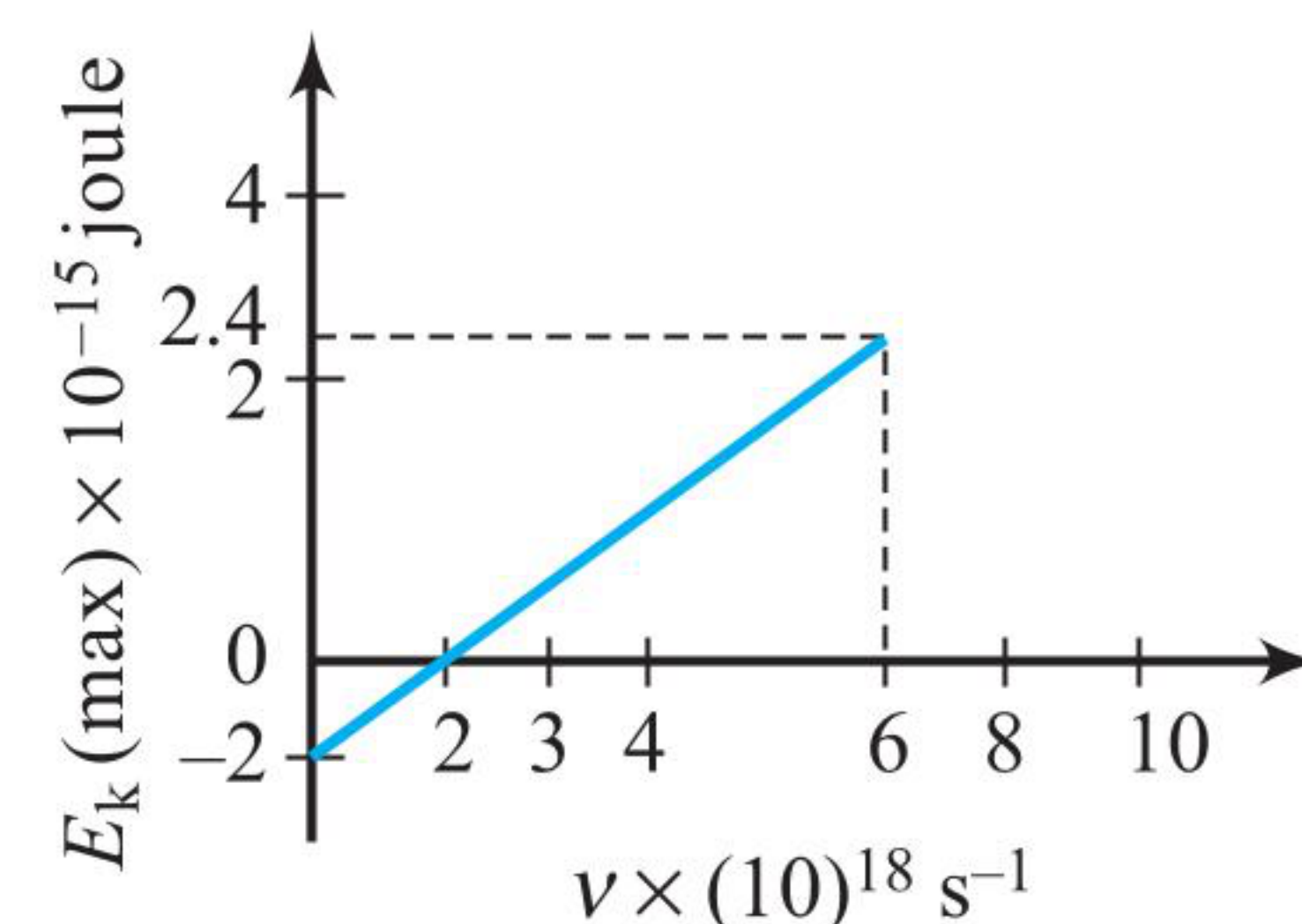
Single Correct Answer Type

- An electron beam accelerated from rest through a potential difference of 5000 V in vacuum is allowed to impinge on a surface normally. The incident current is 50 μA and if the electrons come to rest on striking the surface the force on it is
 (1) $1.1924 \times 10^{-8} \text{ N}$ (2) $2.1 \times 10^{-8} \text{ N}$
 (3) $1.6 \times 10^{-8} \text{ N}$ (4) $1.6 \times 10^{-6} \text{ N}$
- 10^{-3} W of 5000 $\text{\AA}$ light is directed on a photoelectric cell. If the current in the cell is 0.16 μA , the percentage of incident photons which produce photoelectrons, is
 (1) 40% (2) 0.04%
 (3) 20% (4) 10%
- When a certain metallic surface is illuminated with monochromatic light of wavelength λ , the stopping potential for photoelectric current is $3V_0$ and when the same surface is illuminated with light of wavelength 2λ , the stopping potential is V_0 . The threshold wavelength of this surface for photoelectric effect is
 (1) 6λ (2) $4\lambda/3$
 (3) 4λ (4) 8λ
- Threshold frequency for a certain metal is ν_0 . When light of frequency $2\nu_0$ is incident on it, the maximum velocity of photoelectrons is $4 \times 10^8 \text{ cm s}^{-1}$. If frequency of incident radiation is increased to $5\nu_0$, then the maximum velocity of photoelectrons, in cm s^{-1} , will be
 (1) $(4/5) \times 10^8$ (2) 2×10^8
 (3) 8×10^8 (4) 20×10^8
- Light of wavelength 0.6 μm from a sodium lamp falls on a photocell and causes the emission of photoelectrons for which the stopping potential is 0.5 V. With light of wavelength 0.4 μm from a mercury vapor lamp, the stopping potential is 1.5 V. Then, the work function [in electron volts] of the photocell surface is
 (1) 0.75 eV (2) 1.5 eV
 (3) 3 eV (4) 2.5 eV
- Ultraviolet light of wavelength 300 nm and intensity 1.0 W m^{-2} falls on the surface of a photosensitive material. If one per cent of the incident photons produce photoelectrons, then the number of photoelectrons emitted per second from an area of 1.0 cm^2 of the surface is nearly
 (1) $9.61 \times 10^{14} \text{ s}^{-1}$ (2) $4.12 \times 10^{13} \text{ s}^{-1}$
 (3) $1.51 \times 10^{12} \text{ s}^{-1}$ (4) $2.13 \times 10^{11} \text{ s}^{-1}$
- In a photocell, with excitation wavelength λ , the faster electron has speed v . If the excitation wavelength is changed to $3\lambda/4$, the speed of the fastest electron will be
 (1) $v(3/4)^{1/2}$ (2) $v(4/3)^{1/2}$
 (3) less than $v(4/3)^{1/2}$ (4) greater than $v(4/3)^{1/2}$
- Two identical photocathodes receive light of frequencies f_1 and f_2 . If the velocities of the photoelectrons (of mass m) coming out are v_1 and v_2 , respectively, then

$$(1) v_1 - v_2 = \left[\frac{2h}{m} (f_1 - f_2) \right]^{1/2} \quad (2) v_1^2 - v_2^2 = \frac{2h}{m} (f_1 - f_2)$$

$$(3) v_1 + v_2 = \left[\frac{2h}{m} (f_1 - f_2) \right]^{1/2} \quad (4) v_1^2 + v_2^2 = \frac{2h}{m} (f_1 - f_2)$$

- If the intensity of radiation incident on a photocell be increased four times, then the number of photoelectrons and the energy of photoelectrons emitted respectively become
 (1) four times, doubled
 (2) doubled, remains unchanged
 (3) remains unchanged, doubled
 (4) four times, remains unchanged
- If a surface has work function of 3.00 eV, the longest wavelength of light which will cause the emission of electrons is
 (1) $4.8 \times 10^{-7} \text{ m}$ (2) $5.99 \times 10^{-7} \text{ m}$
 (3) $4.13 \times 10^{-7} \text{ m}$ (4) $6.84 \times 10^{-7} \text{ m}$
- In the experiment on photoelectric effect, the graph between $E_k(\text{max})$ and ν is found to be a straight line as shown in figure.



The threshold frequency and Planck's constant according to this graph are

- $3.33 \times 10^{18} \text{ s}^{-1}$, $6 \times 10^{-34} \text{ Js}$
 (2) $6 \times 10^{18} \text{ s}^{-1}$, $6 \times 10^{-34} \text{ Js}$
 (3) $2.66 \times 10^{18} \text{ s}^{-1}$, $4 \times 10^{-34} \text{ Js}$
 (4) $4 \times 10^{18} \text{ s}^{-1}$, $3 \times 10^{-34} \text{ Js}$
- Monochromatic light incident on a metal surface emits electrons with kinetic energies from zero to 2.6 eV. What is the least energy of the incident photon if the tightly bound electron needs 4.2 eV to remove?
 (1) 1.6 eV (2) From 1.6 eV to 6.8 eV
 (3) 6.8 eV (4) More than 6.8 eV
- The work function for sodium surface is 2.0 eV and that for aluminium surface is 4.2 eV. The two metals are illuminated with appropriate radiations so as to cause photoemission. Then
 (1) the threshold frequency for sodium will be less than that for aluminium
 (2) the threshold frequency of sodium will be more than that of aluminium
 (3) both sodium and aluminium will have the same threshold frequency
 (4) none of the above

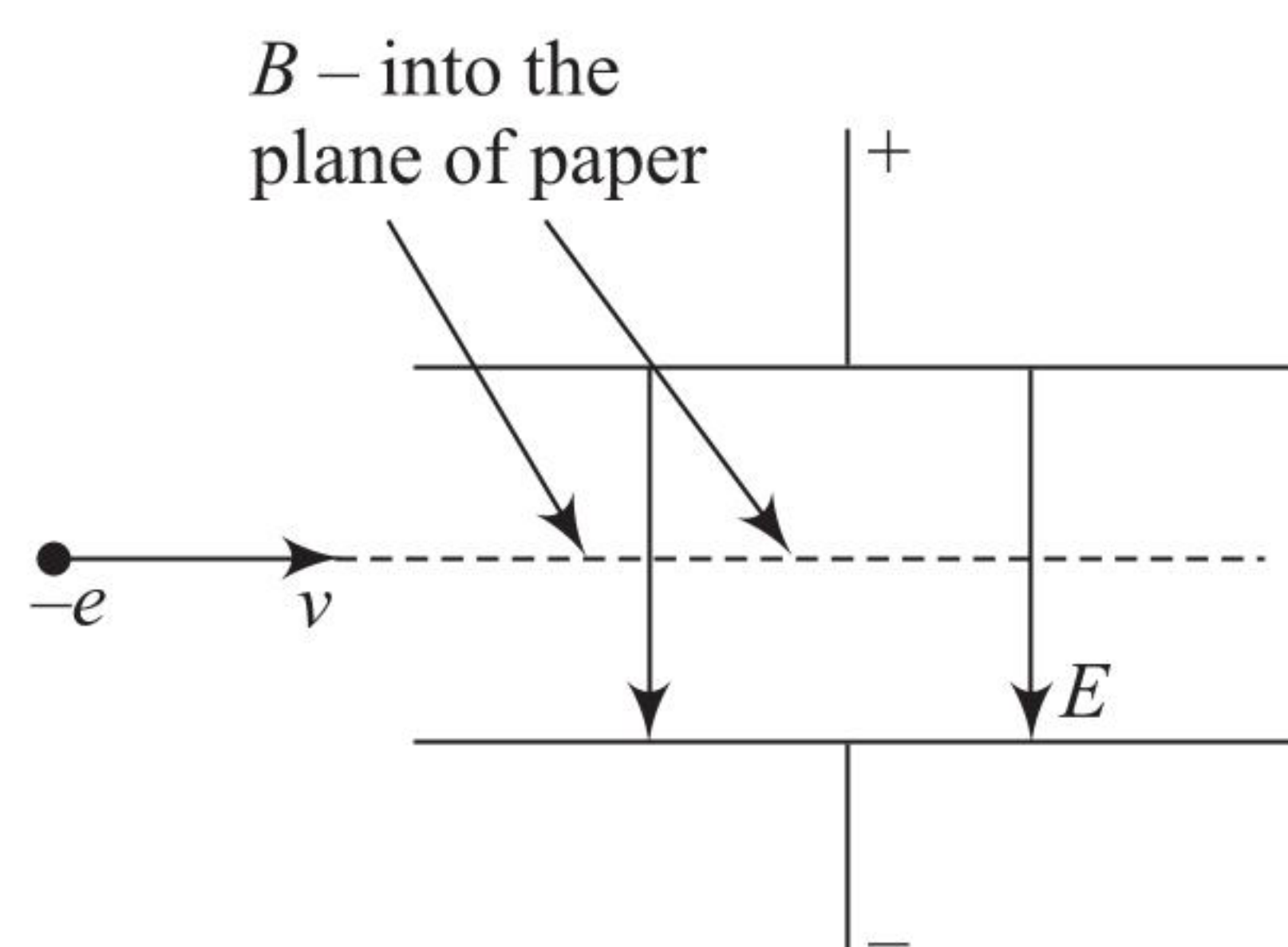
14. Representing the stopping potential V along y -axis and $(1/\lambda)$ along x -axis for a given photocathode, the curve is a straight line, the slope of which is equal to

- (1) e/hc (2) hc/e
(3) ec/h (4) he/c

15. In Q. 14, the intercept on the y -axis is equal to

- (1) $+W/e$ (2) $-W/e$
(3) $-We$ (4) e/W

16. Electrons traveling at a velocity of $2.4 \times 10^6 \text{ m s}^{-1}$ enter a region of crossed electric and magnetic fields as shown in figure. If the electric field is $3.0 \times 10^6 \text{ Vm}$ and the flux density of the magnetic field is 1.5 T , the electrons upon entering the region of the crossed fields will

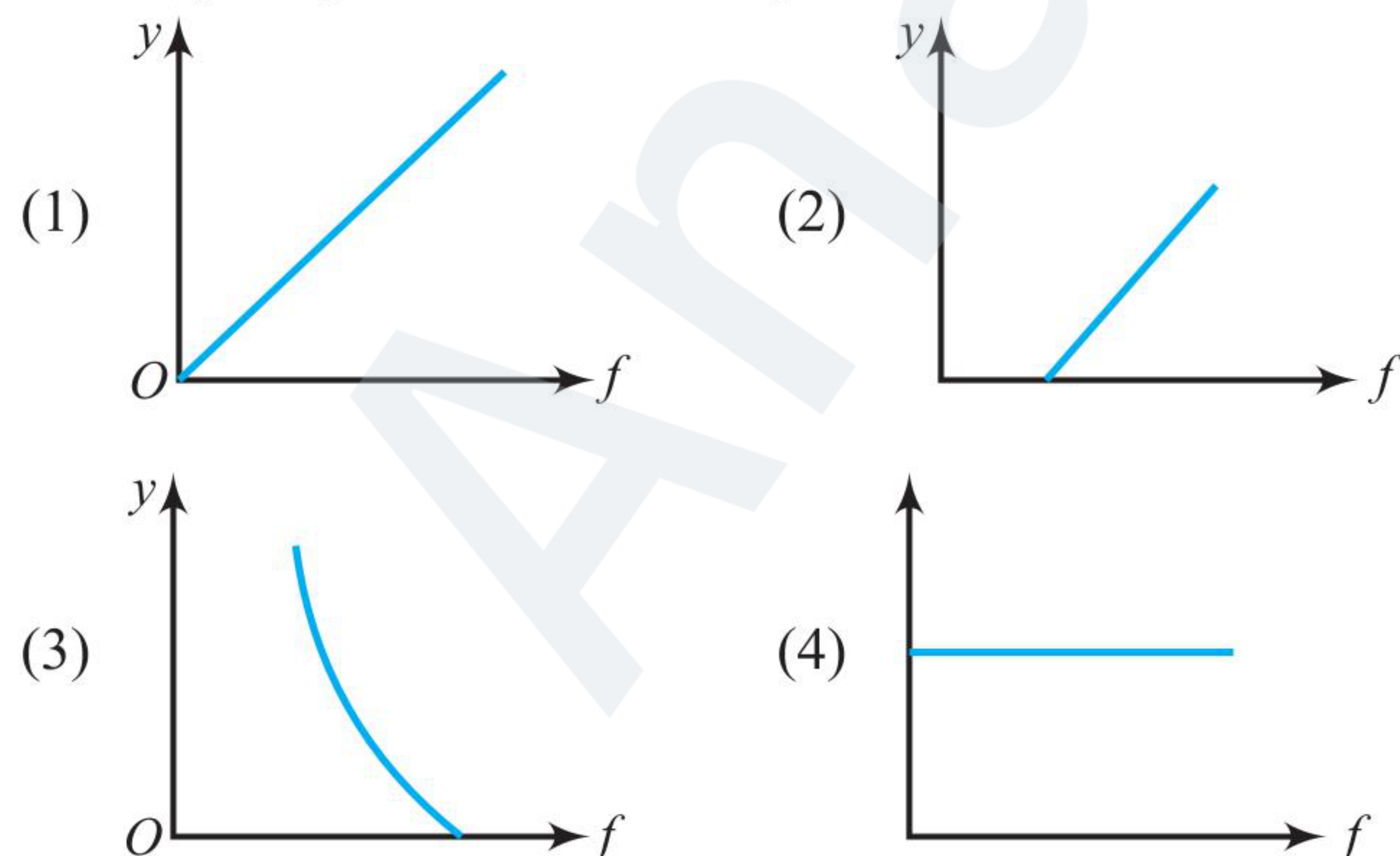


- (1) continue to travel undeflected in their original direction
(2) be deflected upward in the plane of the diagram
(3) be deflected downward on the plane of the diagram
(4) none of the above

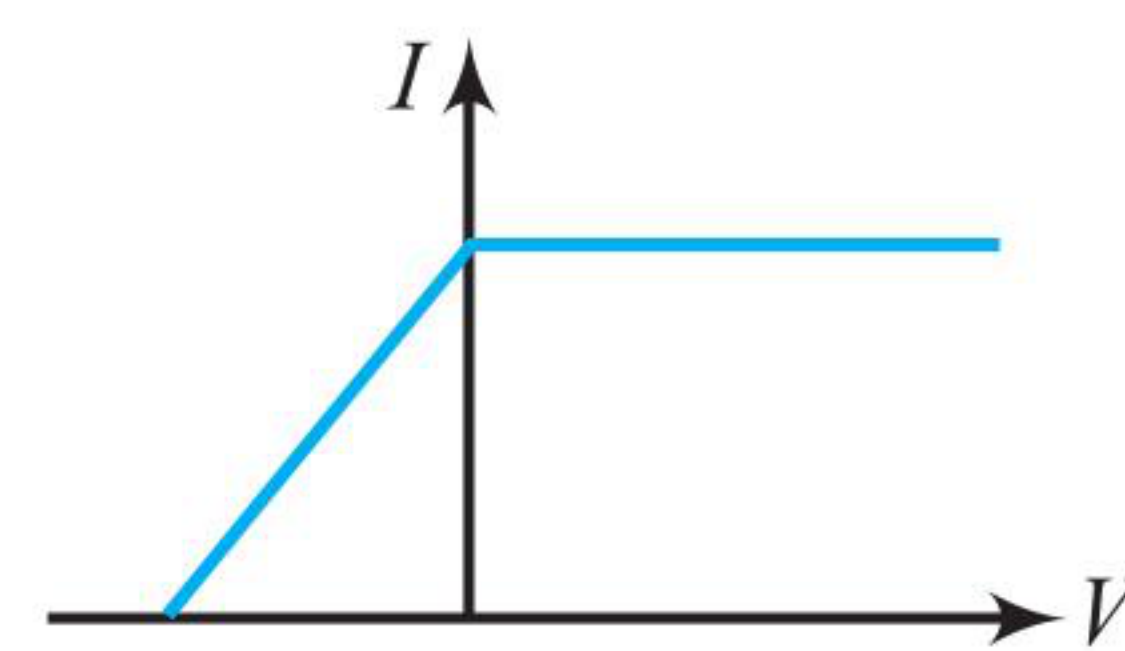
17. A surface irradiated with light $\lambda = 480 \text{ nm}$ gives out electrons with maximum velocity $v \text{ m s}^{-1}$, the cut off wavelength being 600 nm . The same surface would release electrons with maximum velocity $2v \text{ m s}^{-1}$ if it is irradiated by light of wavelength.

- (1) 325 nm (2) 360 nm
(3) 384 nm (4) 300 nm

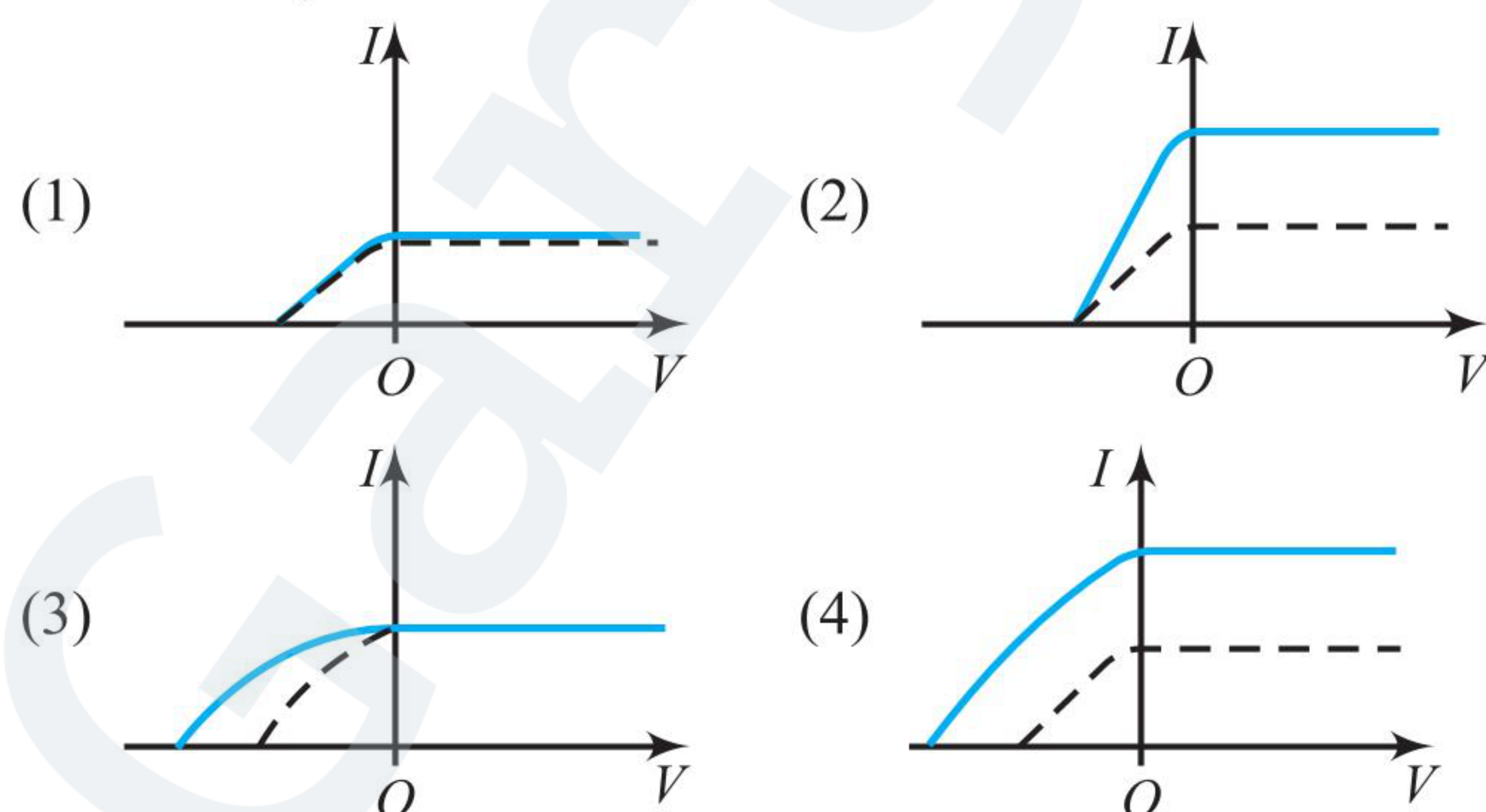
18. In an experiment on the photoelectric effect, an evacuated photocell with a pure metal cathode is used. Which graph best represents the variation of V , the minimum potential difference needed to prevent current from flowing, when x , the frequency of the incident light, is varied?



19. A metal surface in an evacuated tube is illuminated with monochromatic light causing the emission of photoelectrons which are collected at an adjacent electrode. For a given intensity of light, the way in which the photocurrent I depends in the potential difference V between the electrodes is shown by approximate graph in figure.



If the experiment were repeated with light of twice the intensity but the same wavelength, which of the graphs below would best represent the new relation between I and V ? (In these graphs, the result of the original experiment is indicated by a broken line.)



20. If stopping potentials corresponding to wavelengths $4000 \text{ \AA}$ and $4500 \text{ \AA}$ are 1.3 V and 0.9 V , respectively, then the work function of the metal is

- (1) 0.3 eV (2) 1.3 eV
(3) 2.3 eV (4) 5 eV

21. The photoelectric threshold of a certain metal is $3000 \text{ \AA}$. If the radiation of $2000 \text{ \AA}$ is incident on the metal

- (1) electrons will be emitted
(2) positrons will be emitted
(3) protons will be emitted
(4) electrons will not be emitted

22. The frequency and the intensity of a beam of light falling on the surface of a photoelectric material are increased by a factor of two. This will

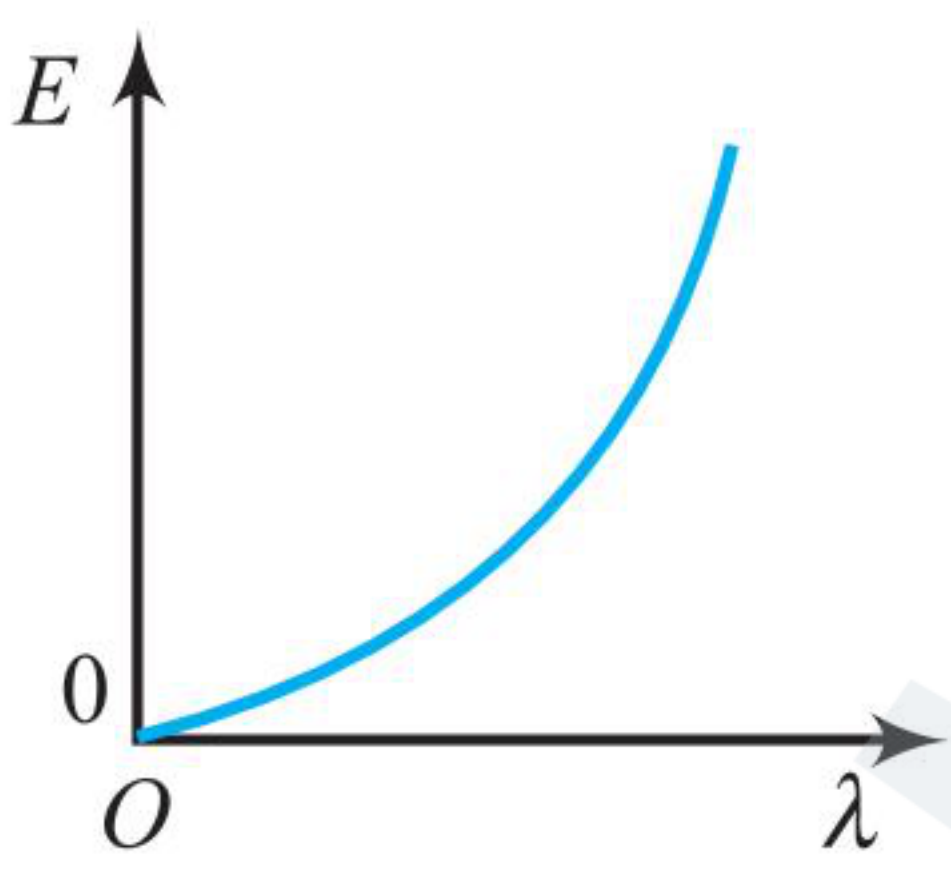
- (1) increase the maximum kinetic energy, the photoelectrons, as well as photoelectric current by a factor of 2
(2) increase the maximum kinetic energy of the photoelectron and would increase the photoelectric current by a factor of 2
(3) increase the maximum kinetic energy of the photoelectrons by a factor of 2 and will have no effect on the magnitude of the photoelectric current produced
(4) not produce any effect on the kinetic energy of the emitted electrons but will increase the photoelectric current by a factor of 2

23. The frequency of incident light falling on a photosensitive metal plate is doubled, the KE of the emitted photoelectrons is

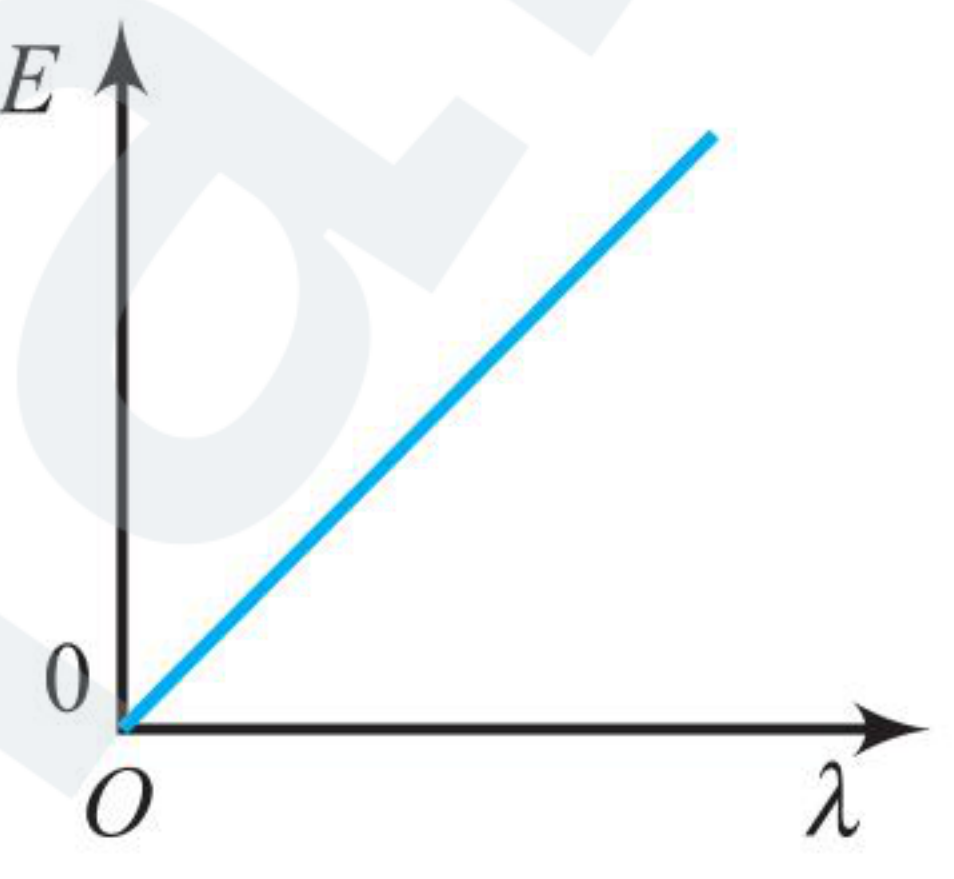
- (1) double the earlier value (2) unchanged
(3) more than doubled (4) less than doubled

24. Lights of two different frequencies whose photons have energies 1 eV and 2.5 eV , respectively, successively illuminate a metal whose work function is 0.5 eV . The ratio of the maximum speeds of the emitted electrons will be

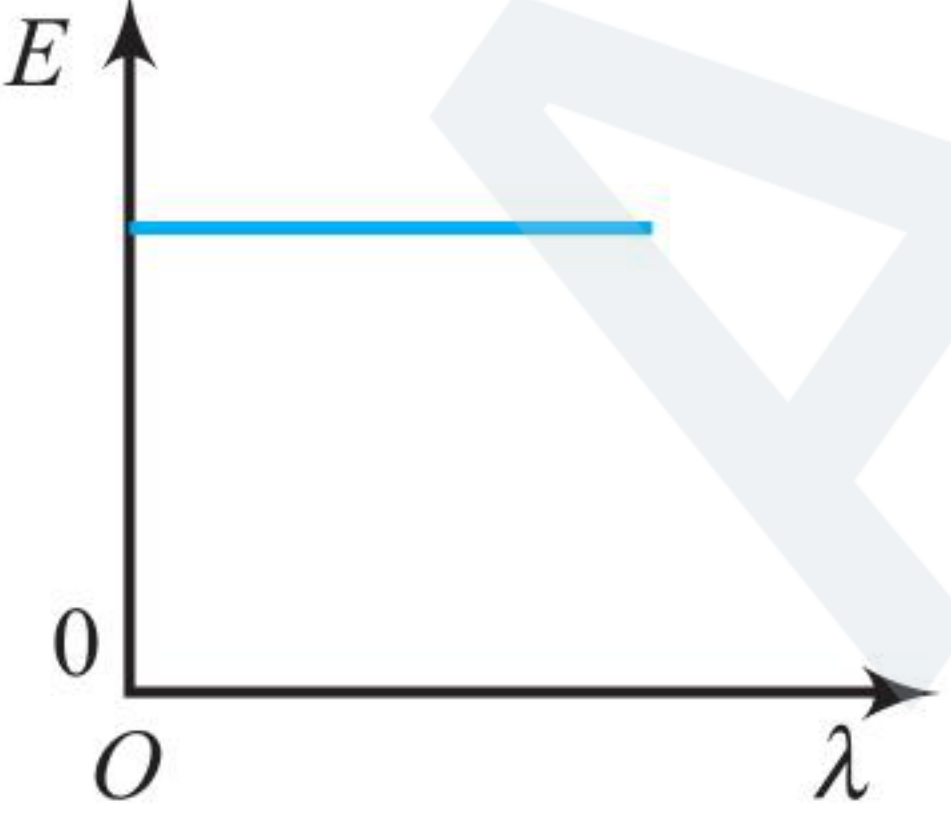
- (1) 1:5 (2) 1:4
(3) 1:2 (4) 1:1
25. A proton when accelerated through a potential difference of V volt has a wavelength λ associated with it. An α -particle in order to have the same λ must be accelerated through a potential difference of
(1) V volt (2) $4V$ volt
(3) $2V$ volt (4) $(V/8)$ volt
26. A cesium photocell, with a steady potential difference of 60 V across it, is illuminated by a small bright light placed 1 m away. When the same light is placed 2 m away, the electrons crossing the photocell
(1) each carry one-quarter of their previous momentum
(2) each carry one-quarter of their previous energy
(3) are one-quarter as numerous
(4) are half as numerous
27. If 5% of the energy supplied to a bulb is irradiated as visible light, how many quanta are emitted per second by a 100 W lamp? Assume wavelength of visible light as 5.6×10^{-5} cm.
(1) 1.4×10^{19} (2) 3×10^3
(3) 1.4×10^{-19} (4) 3×10^4
28. Five volt of stopping potential is needed for the photoelectrons emitted out of a surface of work function 2.2 eV by the radiation of wavelength
(1) 1719 Å (2) 3444 Å
(3) 861 Å (4) 3000 Å
29. The work functions for tungsten and sodium are 4.5 eV and 2.3 eV, respectively. If the threshold wavelength λ for sodium is 5460 Å, the value of λ for tungsten is
(1) 5893 Å (2) 10683 Å
(3) 2791 Å (4) 528 Å
30. The photoelectric threshold for some material is 200 nm. The material is irradiated with radiations of wavelength 40 nm. The maximum kinetic energy of the emitted photoelectrons is
(1) 2 eV (2) 1 eV
(3) 0.5 eV (4) none of these
31. Photoelectric work function of a metal is 1 eV. Light of wavelength $\lambda = 3000$ Å falls on it. The photoelectrons come out with velocity
(1) 10 m s^{-1} (2) 10^3 m s^{-1}
(3) 10^4 m s^{-1} (4) 10^6 m s^{-1}
32. If a surface has a work function 4.0 eV, what is the maximum velocity of electrons liberated from the surface when it is irradiated with ultraviolet radiation of wavelength 0.2 μm ?
(1) $4.4 \times 10^5 \text{ m s}^{-1}$ (2) $8.8 \times 10^7 \text{ m s}^{-1}$
(3) $8.8 \times 10^5 \text{ m s}^{-1}$ (4) $4.4 \times 10^7 \text{ m s}^{-1}$
33. A modern 200 W sodium street lamp emits yellow light of wavelength 0.6 μm . Assuming it to be 25% efficient in converting electrical energy to light, the number of photons of yellow light it emits per second is
(1) 62×10^{20} (2) 3×10^{19}
(3) 1.5×10^{20} (4) 6×10^{18}
34. An electron of mass m_e and a proton of mass m_p are accelerated through the same potential difference. The ratio of the de Broglie wavelength associated with an electron to that associated with proton is
(1) 1 (2) m_p/m_e
(3) m_e/m_p (4) $\sqrt{m_p/m_e}$
35. A material particle with a rest mass m_0 is moving with a velocity of light c . Then, the wavelength of the de Broglie wave associated with it is
(1) $(h/m_0 c)$ (2) zero
(3) ∞ (4) $(m_0 c/h)$
36. If a photocell is illuminated with a radiation of 1240 Å, then stopping potential is found to be 8 V. The work function of the emitter and the threshold wavelength are
(1) 1 eV, 5200 Å (2) 2 eV, 6200 Å
(3) 3 eV, 7200 Å (4) 4 eV, 4200 Å
37. Silver has a work function of 4.7 eV. When ultraviolet light of wavelength 100 nm is incident upon it, a potential of 7.7 V is required to stop the photoelectrons from reaching the collector plate. How much potential will be required to stop the photoelectrons when light of wavelength 200 nm is incident upon silver?
(1) 1.5 V (2) 3.85 V
(3) 2.35 V (4) 15.4 V
38. Out of a photon and an electron, the equation $E = pc$, is valid for
(1) both (2) neither
(3) photon only (4) electron only
39. When a centimeter thick surface is illuminated with light of wavelength λ , the stopping potential is V . When the same surface is illuminated by light of wavelength 2λ , the stopping potential is $V/3$. Threshold wavelength for the metallic surface is
(1) $4\lambda/3$ (2) 4λ
(3) 6λ (4) $8\lambda/3$
40. Light of wavelength λ strikes a photoelectric surface and electrons are ejected with kinetic energy K . If K is to be increased to exactly twice its original value, the wavelength must be changed to λ' such that
(1) $\lambda' < \lambda/2$ (2) $\lambda' > \lambda/2$
(3) $\lambda > \lambda' > \lambda/2$ (4) $\lambda' = \lambda/2$
41. The ratio of momenta of an electron and an α -particle which are accelerated from rest by a potential difference of 100 V is
(1) 1 (2) $\sqrt{2m_e/m_\alpha}$
(3) $\sqrt{m_e/m_\alpha}$ (4) $\sqrt{m_e/2m_\alpha}$
42. The kinetic energy of an electron is E when the incident wavelength is λ . To increase the KE of the electron to $2E$, the incident wavelength must be
(1) 2λ
(2) $\lambda/2$
(3) $(hc\lambda)/(E\lambda + hc)$
(4) $(hc)/(E\lambda + hc)$

43. The KE of the photoelectrons is E when the incident wavelength is $\lambda/2$. The KE becomes $2E$ when the incident wavelength is $\lambda/3$. The work function of the metal is
 (1) hc/λ (2) $2hc/\lambda$
 (3) $3hc/\lambda$ (4) $hc/3\lambda$
44. If λ_0 stands for mid-wavelength in the visible region, the de Broglie wavelength for 100 V electrons is nearest to
 (1) $\lambda_0/5$ (2) $\lambda_0/50$
 (3) $\lambda_0/500$ (4) $\lambda_0/5000$
45. When a metallic surface is illuminated by a light of frequency 8×10^{14} Hz, photoelectron of maximum energy 0.5 eV is emitted. When the same surface is illuminated by light of frequency 12×10^{14} Hz, photoelectron of maximum energy 2 eV is emitted. The work function is
 (1) 0.5 eV (2) 2.85 eV
 (3) 2.5 eV (4) 3.5 eV
46. The de Broglie wavelength of neutrons in thermal equilibrium is (given $m_n = 1.6 \times 10^{-27}$ kg)
 (1) $30.8/\sqrt{T}$ Å (2) $3.08/\sqrt{T}$ Å
 (3) $0.308/\sqrt{T}$ Å (4) $0.0308/\sqrt{T}$ Å
47. A particle of mass M at rest decays into two masses m_1 and m_2 with non-zero velocities. The ratio λ_1/λ_2 of de Broglie wavelengths of the particles is
 (1) m_2/m_1 (2) m_1/m_2
 (3) $\sqrt{m_1}/\sqrt{m_2}$ (4) 1 : 1
48. If λ_1 and λ_2 denote the wavelengths of de Broglie waves for electrons in the first and second Bohr orbits in a hydrogen atom, then λ_1/λ_2 is equal to
 (1) 2/1 (2) 1/2
 (3) 1/4 (4) 4/1
49. Which curve shows the relationship between the energy E and the wavelength λ of a photon of electromagnetic radiation?
- 

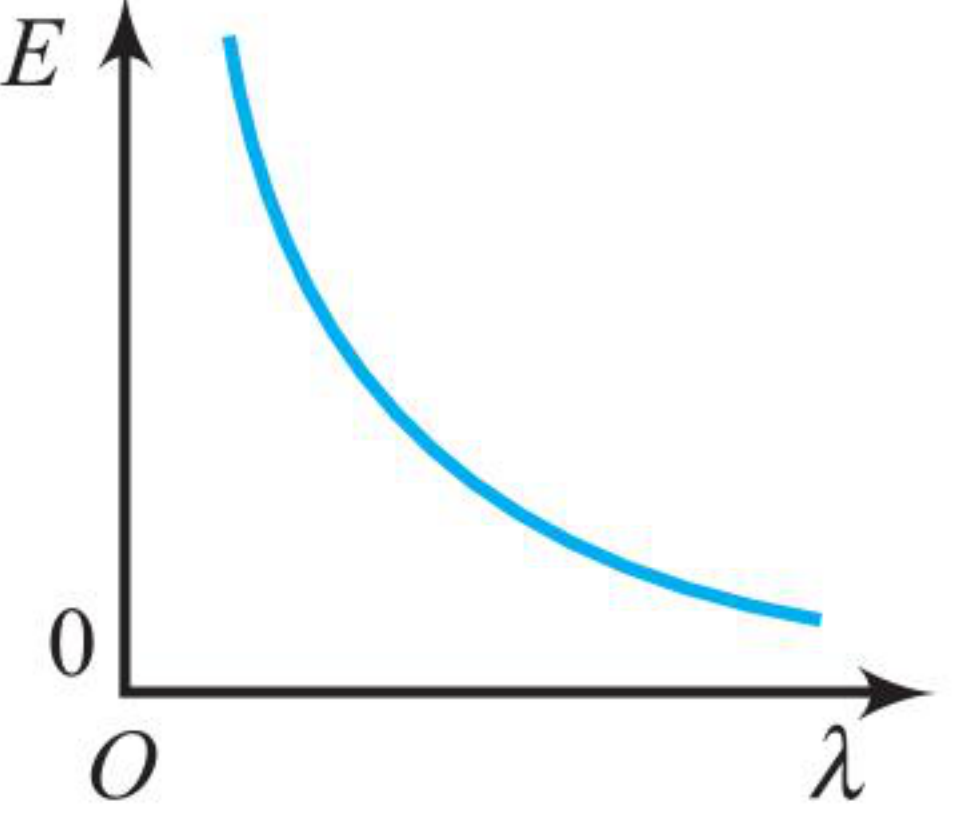
(1)



(2)

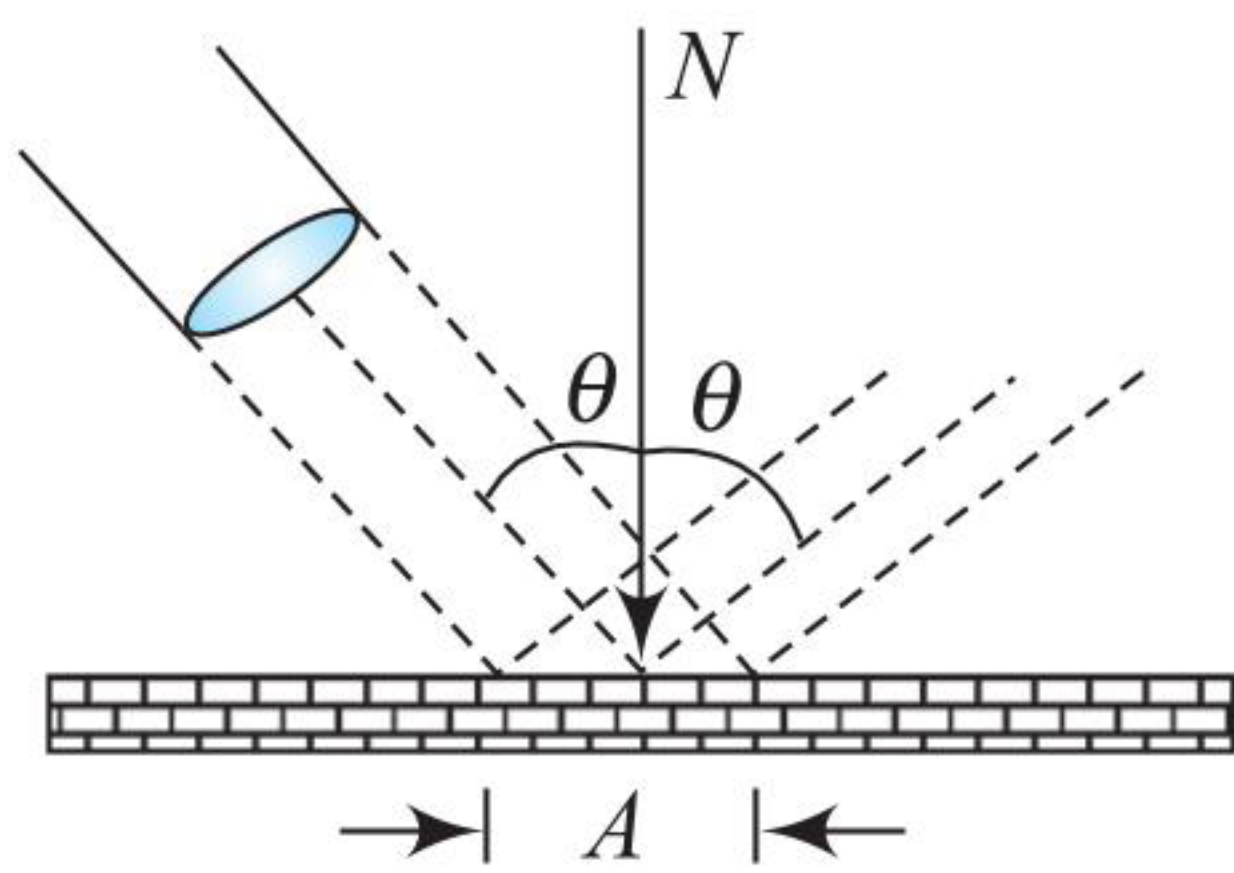


(3)



(4)
50. Work function of nickel is 5.01 eV. When ultraviolet radiation of wavelength 200 Å is incident on it, electrons are emitted. What will be the maximum velocity of emitted electrons?
 (1) 3×10^8 m s⁻¹ (2) 6.46×10^5 m s⁻¹
 (3) 10.36×10^5 m s⁻¹ (4) 8.54×10^6 m s⁻¹
51. An electron is accelerated through a potential difference of V volt. It has a wavelength λ associated with it. Through what potential difference an electron must be accelerated so that its de Broglie wavelength is the same as that of a proton? Take mass of proton to be 1837 times larger than the mass of electron.
 (1) V volt (2) $1837V$ volt
 (3) $V/1837$ volt (4) $\sqrt{1837} V$ volt
52. The kinetic energy of most energetic electrons emitted from a metallic surface is doubled when the wavelength λ of the incident radiation is changed from 400 nm to 310 nm. The work function of the metal is
 (1) 0.9 eV (2) 1.7 eV
 (3) 2.2 eV (4) 3.1 eV
53. Two identical metal plates show photoelectric effect. Light of wavelength λ_A falls on plate A and λ_B falls on plate B , $\lambda_A = 2\lambda_B$. The maximum KE of the photoelectrons are K_A and K_B , respectively. Which one of the following is true?
 (1) $2K_A = K_B$ (2) $K_A = 2K_B$
 (3) $K_A < K_B/2$ (4) $K_A > 2K_B$
54. The potential difference applied to an X-ray tube is V . The ratio of the de Broglie wavelength of electron to the minimum wavelength of X-ray is directly proportional to
 (1) V (2) $\sqrt{V}$
 (3) $V^{3/2}$ (4) $V^{7/2}$
55. The maximum velocity of electrons emitted from a metal surface is v . What would be the maximum velocity if the frequency of incident light is increased by a factor of 4?
 (1) $2v$ (2) $> 2v$
 (3) $< 2v$ (4) between $2v$ and $4v$
56. The eye can detect 5×10^4 photons (m² s)⁻¹ of green light ($\lambda = 5000$ Å), while ear can detect 10^{-13} W m⁻². As a power detector, which is more sensitive and by what factor?
 (1) Eye is more sensitive by a factor of 5.00.
 (2) Ear is more sensitive by a factor of 5.00.
 (3) Both are equally sensitive.
 (4) Eye is more sensitive by a factor of 10^{-1} .
57. A photon of wavelength 0.1 Å is emitted by a helium atom as a consequence of the emission of photon. The KE gained by helium atom is
 (1) 0.05 eV (2) 1.05 eV
 (3) 2.05 eV (4) 3.05 eV
58. How many photons of a radiation of wavelength $\lambda = 5 \times 10^{-7}$ m must fall per second on a blackened plate in order to produce a force of 6.62×10^{-5} N?
 (1) 3×10^{19} (2) 5×10^{22}
 (3) 2×10^{22} (4) 1.67×10^{18}
59. A nozzle throws a stream of gas against a wall with a velocity v much larger than the thermal agitation of the molecules. The wall deflects the molecules without changing the magnitude of their velocity. Also, assume that the force exerted on the

wall by the molecules is perpendicular to the wall. (This is not strictly true for a rough wall). Find the force exerted on the wall.

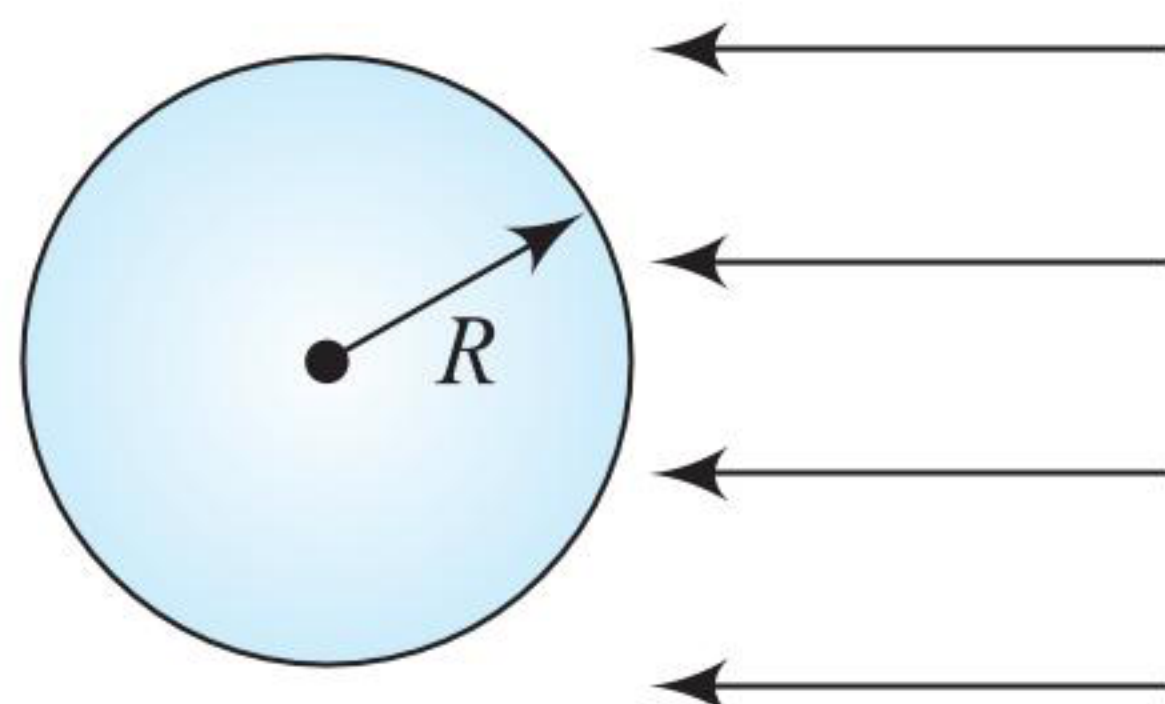


- (1) $Anmv^2 \cos^2 \theta$ (2) $2Anmv^2 \cos^2 \theta$
 (3) $2Anmv^2 \sin^2 \theta$ (4) $Anmv^2 \cos \theta$

60. A plane light wave of intensity $I = 0.20 \text{ W cm}^{-2}$ falls on a plane mirror surface with reflection coefficient $\rho = 0.8$. The angle of incidence is 45° . In terms of corpuscular theory, find the magnitude of the normal pressure exerted on that surface.

- (1) 1.2 N cm^{-2} (2) 0.2 N cm^{-2}
 (3) 2.6 N cm^{-2} (4) 0.5 N cm^{-2}

61. A plane wave of intensity $I = 0.70 \text{ W cm}^{-2}$ illuminates a sphere with ideal mirror surface. The radius of sphere is $R = 5.0 \text{ cm}$. From the standpoint of photon theory, find the force that light exerts on the sphere.



- (1) $0.8 \mu\text{N}$ (2) $0.2 \mu\text{N}$
 (3) $0.5 \mu\text{N}$ (4) $1.2 \mu\text{N}$

62. Light of wavelength λ from a small 0.5 mW He-Ne laser source, used in the school laboratory, shines from a spacecraft of mass 1000 kg . Estimate the time needed for the spacecraft to reach a velocity of 1.0 km s^{-1} from rest. The momentum p of a photon of wavelength λ is given by $p = h/\lambda$, where h is Planck's constant.

- (1) 6×10^{18} (2) 3×10^{17}
 (3) 6×10^{17} (4) 2×10^{15}

63. What is the de Broglie wavelength of the wave associated with an electron that has been accelerated through a potential difference of 50.0 V ?

- (1) 2.7×10^{-10} (2) 1.74×10^{-10}
 (3) 3.6×10^{-9} (4) 4.9×10^{-11}

64. An α -particle and a proton are fired through the same magnetic field which is perpendicular to their velocity vectors. The α -particle and the proton move such that radius of curvature of their paths is same. Find the ratio of their de Broglie wavelengths.

- (1) 2:3 (2) 3:4
 (3) 5:7 (4) 1:2

65. Find the ratio of de Broglie wavelength of a proton and an α -particle which have been accelerated through same potential difference.

- (1) $2\sqrt{2}:1$ (2) 3:2
 (3) $3\sqrt{2}:1$ (4) 2:1

66. All electrons ejected from a surface by incident light of wavelength 200 nm can be stopped before traveling 1 m in the direction of a uniform electric field of 4 NC^{-1} . The work function of the surface is

- (1) 4 eV (2) 6.2 eV
 (3) 2 eV (4) 2.2 eV

67. If the short wavelength limit of the continuous spectrum coming out of a Coolidge tube is $10 \text{ \AA}$, then the de Broglie wavelength of the electrons reaching the target metal in the Coolidge tube is approximately

- (1) $0.3 \text{ \AA}$ (2) $3 \text{ \AA}$
 (3) $30 \text{ \AA}$ (4) $10 \text{ \AA}$

68. In a photoelectric effect, electrons are emitted

- (1) with a maximum velocity proportional to the frequency of the incident radiation
 (2) at a rate that is independent of the intensity of the incident radiation
 (3) only if the frequency of the incident radiation is above a certain threshold value
 (4) only if the temperature of the emitter is high

69. A 60 W bulb is placed at a distance of 4 m from you. The bulb is emitting light of wavelength 600 nm uniformly in all directions. In 0.1 s , how many photons enter your eye if the pupil of the eye is having a diameter of 2 mm ? [Take $hc = 1240 \text{ eV-nm}$]

- (1) 2.84×10^{12} (2) 2.84×10^{11}
 (3) 9.37×10^{11} (4) 6.48×10^{11}

70. Two identical non-relativistic particles A and B move at right angles to each other, possessing de Broglie wavelengths λ_1 and λ_2 , respectively. The de Broglie wavelength of each particle in their centre of mass frame of reference is

- (1) $\lambda_1 + \lambda_2$ (2) $2\lambda_1\lambda_2 / (\sqrt{\lambda_1^2 + \lambda_2^2})$
 (3) $\lambda_1\lambda_2 / (\sqrt{|\lambda_1^2 + \lambda_2^2|})$ (4) $(\lambda_1 + \lambda_2)/2$

71. A sodium metal piece is illuminated with light of wavelength $0.3 \mu\text{m}$. The work function of sodium is 2.46 eV . For this situation, mark out the correct statement(s).

- (1) The maximum kinetic energy of the ejected photoelectrons is 1.68 eV
 (2) The cut-off wavelength for sodium is 505 nm
 (3) The minimum photon energy of incident light for photoelectric effect to take place is 2.46 eV
 (4) All of the above

72. The kinetic energy of a particle is equal to the energy of a photon. The particle moves at 5% of the speed of light. The ratio of the photon wavelength to the de Broglie wavelength of the particle is

[No need to use relativistic formula for the particle.]

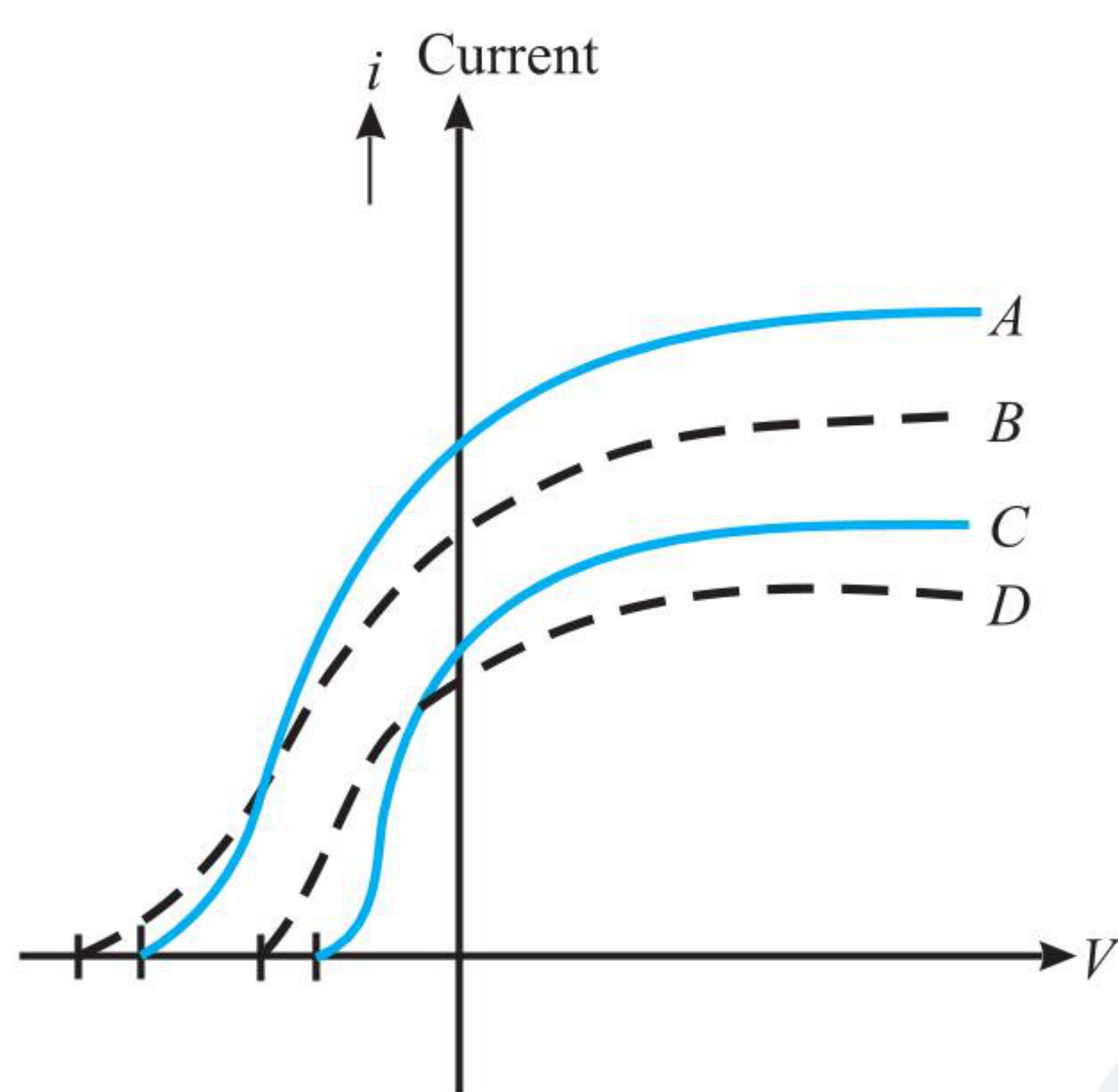
- (1) 40 (2) 4
 (3) 2 (4) 80

73. The resolving power of an electron microscope operated at 16 kV is R . The resolving power of the electron microscope when operated at 4 kV is
 (1) $R/4$ (2) $R/2$
 (3) $4R$ (4) $2R$
74. Two electrons are moving with same speed v . One electron enters a region of uniform electric field while the other enters a region of uniform magnetic field, then after some time de Broglie wavelengths of two are λ_1 and λ_2 , respectively. Now,
 (1) $\lambda_1 = \lambda_2$ (2) $\lambda_1 > \lambda_2$
 (3) $\lambda_1 < \lambda_2$ (4) λ_1 can be greater than or less than λ_2
75. The energy of a photon is equal to the kinetic energy of a proton. The energy of photon is E . Let λ_1 be the de Broglie wavelength of the proton and λ_2 be the wavelength of the photon. Then, λ_1/λ_2 is proportional to
 (1) E^0 (2) $E^{1/2}$
 (3) E^{-1} (4) E^{-2}
76. Up to what potential V can a zinc ball (work function 3.74 eV) removed from other bodies be charged by irradiating it with light of $\lambda = 200$ nm?
 (1) 2.5 V (2) 1.8 V
 (3) 2.2 V (4) 3 V
77. A sensor is exposed for time t to a lamp of power P placed at a distance l . The sensor has an opening that is $4d$ in diameter. Assuming all energy of the lamp is given off as light, the number of photons entering the sensor if the wavelength of light is λ is
 (1) $N = P\lambda d^2 t / hcl^2$ (2) $N = 4P\lambda d^2 t / hcl^2$
 (3) $N = P\lambda d^2 t / 4hcl^2$ (4) $N = P\lambda d^2 t / 16hcl^2$
78. A photon has same wavelength as the de Broglie wavelength of electrons. Given c = speed of light, v = speed of electron. Which of the following relation is correct? [Here E_e = kinetic energy of electron, E_{ph} = energy of photon, p_e = momentum of electron and p_{ph} = momentum of photon]
 (1) $E_e/E_{ph} = 2c/v$ (2) $E_e/E_{ph} = v/2c$
 (3) $p_e/p_{ph} = 2c/v$ (4) $p_e/p_{ph} = c/v$
79. A particle of mass $3m$ at rest decays into two particles of masses m and $2m$ having non-zero velocities. The ratio of the de Broglie wavelengths of the particles (λ_1/λ_2) is
 (1) $1/2$ (2) $1/4$
 (3) 2 (4) none of these
80. The radius of the second orbit of an electron in hydrogen atom is $2.116 \text{ \AA}$. The de Broglie wavelength associated with this electron in this orbit would be
 (1) $6.64 \text{ \AA}$ (2) $1.058 \text{ \AA}$
 (3) $2.116 \text{ \AA}$ (4) $13.28 \text{ \AA}$
81. Radiation of wavelength 546 nm falls on a photo cathode and electrons with maximum kinetic energy of 0.18 eV are emitted. When radiation of wavelength 185 nm falls on the same surface, a (negative) stopping potential of 4.6 V has to be applied to the collector cathode to reduce the photoelectric current to zero. Then, the ratio h/e is
 (1) $6.6 \times 10^{-15} \text{ J s C}^{-1}$ (2) $4.12 \times 10^{-15} \text{ J s C}^{-1}$
 (3) $6.6 \times 10^{-34} \text{ J s C}^{-1}$ (4) $4.12 \times 10^{-34} \text{ J s C}^{-1}$
82. The human eye is most sensitive to green light of wavelength 505 nm . Experiments have found that when people are kept in a dark room until their eyes adapt to the darkness, a single photon of green light will trigger receptor cells in the rods of the retina. The velocity of typical bacterium of mass $9.5 \times 10^{-12} \text{ g}$, if it had absorbed all energy of photon, is nearly
 (1) 10^{-6} m s^{-1} (2) 10^{-8} m s^{-1}
 (3) $10^{-10} \text{ m s}^{-1}$ (4) $10^{-13} \text{ m s}^{-1}$
83. The light sensitive compound on most photographic films is silver bromide AgBr. A film is exposed when the light energy absorbed dissociates this molecule into its atoms. The energy of dissociation of AgBr is 10^5 J mol^{-1} . For a photon that is just able to dissociate a molecule of AgBr, the photon energy is
 (1) 1.04 eV (2) 2.08 eV
 (3) 3.12 eV (4) 4.16 eV

Multiple Correct Answers Type

- Photoelectric effect supports the quantum nature of light because
 - there is a minimum frequency of light below which no photoelectrons are emitted
 - the maximum KE of photoelectrons depends only on the frequency of light and not on its intensity
 - even when the metal surface is faintly illuminated by light of wavelength less than the threshold wavelength, the photoelectrons leave the surface immediately
 - electric charge of photoelectrons is quantized
- In a photoelectric experiment, the wavelength of the incident light is decreased from $6000 \text{ \AA}$ to $4000 \text{ \AA}$. While the intensity of radiations remains the same,
 - the cut-off potential will decrease
 - the cut-off potential will increase
 - the photoelectric current will increase
 - the kinetic energy of the emitted electrons will increase
- When photons of energy hc/λ fall on a metal surface, photoelectrons are ejected from it. If the work function of the surface is $h\nu_0$, then
 - maximum kinetic energy of the electron is $[(hc/\lambda) - h\nu_0]$
 - maximum kinetic energy of the photoelectron is equal to (hc/λ)
 - minimum KE of the photoelectron is zero
 - minimum kinetic energy of the photoelectron is equal to hc/λ
- Threshold wavelength of certain metal is λ_0 . A radiation of wavelength $\lambda < \lambda_0$ is incident on the plate. Then, choose the correct statement from the following.

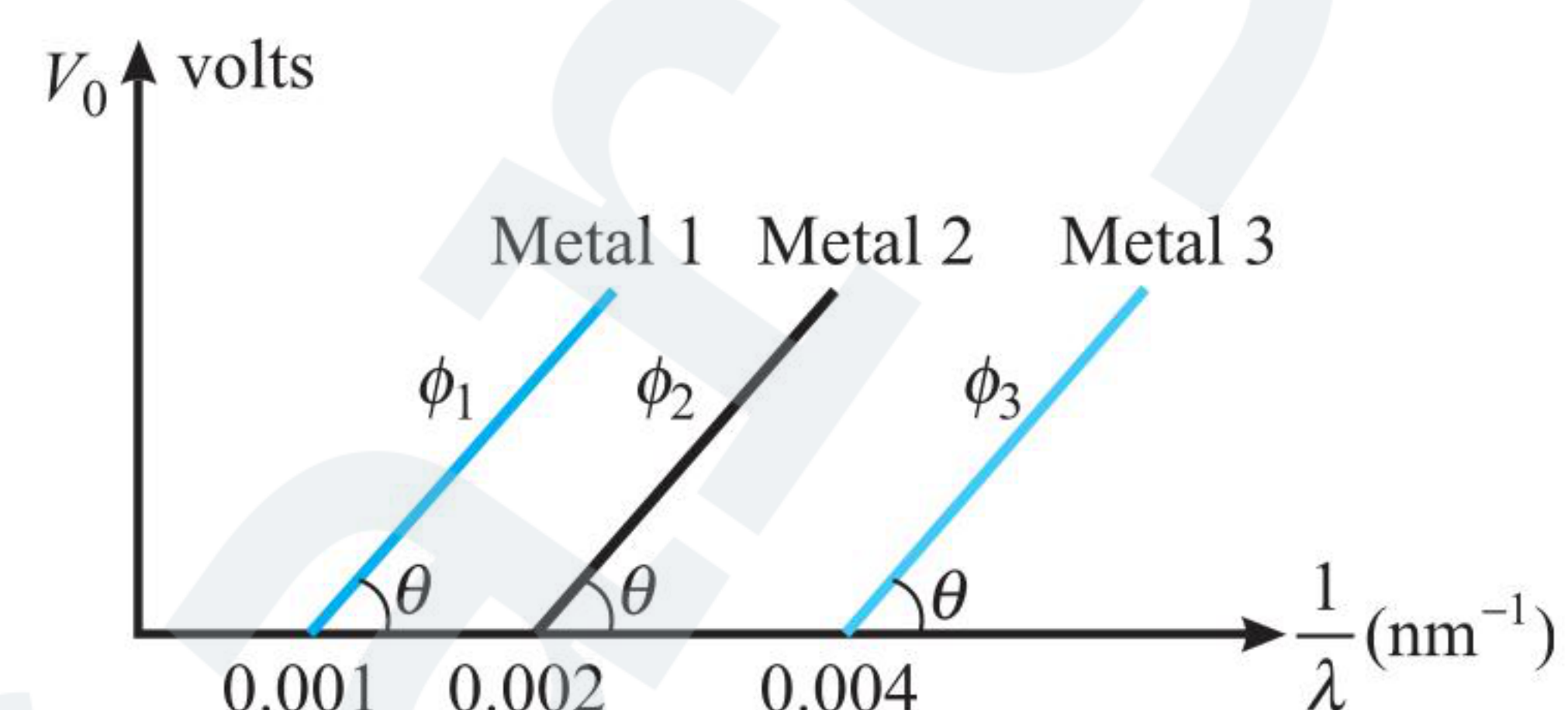
- (1) Initially electrons will come out from the plate
 (2) The ejected electrons experience retarding force due to development of positive charges on the plate
 (3) After some time, ejection of electrons stops
 (4) None of the above
5. Photons of wavelength 248 nm falls on a metal surface whose work function is 2.2 eV. Assume thru each photoelectron inside the metal lattice may come out of the surface or collide with the lattice before coming out. In each collision with the lattice, it loses 20% of its existing energy. Which of the following can be a kinetic energy of an ejected photoelectron?
- (1) 2.8 eV (2) 2.24 eV
 (3) 1.8 eV (4) 1 eV
6. The figure shows the results of an experiment involving photoelectric effect. The graphs *A*, *B*, *C* and *D* relate to a light beam having different wavelengths. Select the correct alternative



- (1) Beam *B* has highest frequency
 (2) Beam *C* has longest wavelength
 (3) Beam *A* has highest rate of photoelectric emission
 (4) Photoelectrons emitted by *B* have highest momentum
7. The threshold wavelength for photoelectric emission from a material is 5200 Å. Photoelectrons will be emitted when this material is illuminated with monochromatic radiation from a
- (1) 50 W infrared lamp
 (2) 1 W infrared lamp
 (3) 50 W ultraviolet lamp
 (4) 1 W ultraviolet lamp
8. When a monochromatic point source of light is at a distance of 0.2 m from a photoelectric cell, the cut-off voltage and the saturation current are, respectively, 0.6 V and 18.0 mA. If the same source is placed 0.6 m away from the photoelectric cell, then
- (1) the stopping potential will be 0.2 V
 (2) the stopping potential will be 0.6 V
 (3) the saturation current will be 6.0 mA
 (4) the saturation current will be 2.0 mA
9. When photons of energy 4.25 eV strike the surface of metal *A*, the ejected photoelectrons have maximum kinetic energy T_A and de Broglie wavelength λ_A . The maximum kinetic energy of photoelectrons liberated

from another metal *B* by photons of energy 4.70 eV is $T_B = (T_A \times 1.50)$ eV. If the de Broglie wavelength of these photoelectrons is $\lambda_B = 2\lambda_A$, then

- (1) the work function of *A* is 2.25 eV
 (2) the work function of *B* is 4.20 eV
 (3) $T_A = 2.00$ eV
 (4) $T_B = 2.75$ eV
10. The graph between the stopping potential (V_0) and $(1/\lambda)$ is shown in the figure. ϕ_1 , ϕ_2 and ϕ_3 are work functions. Then, which of the following is/are correct?



- (1) $\phi_1 : \phi_2 : \phi_3 = 1 : 2 : 4$
 (2) $\phi_1 : \phi_2 : \phi_3 = 4 : 2 : 1$
 (3) $\tan \theta \propto (hc)/e$
 (4) Ultraviolet light can be used to emit photoelectrons from metal 2 and metal 3 only
11. A surface has light of wavelength $\lambda_1 = 550$ nm incident on it, causing the ejection of photoelectrons for which the stopping potential is $V_{s1} = 0.19$ V. If the radiation of wavelength $\lambda_2 = 190$ nm is now incident on the surface,
- (1) the stopping potential is $V_{s2} = 4.47$ V
 (2) the stopping potential is $V_{s2} = 6.54$ V
 (3) the work function of the surface is 2.07 eV
 (4) the threshold frequency for the surface is 5×10^{14} Hz
12. A uniform monochromatic beam of light of wavelength 365×10^{-9} m and intensity 10^{-8} W m $^{-2}$ falls on a surface having absorption coefficient 0.8 and work function 1.6 eV.
- (1) the number of electrons emitted per square meter per second are 1.47×10^{10} m $^{-2}$ s $^{-1}$
 (2) the number of electrons emitted per square meter per second are 3.75×10^{10} m $^{-2}$ s $^{-1}$
 (3) power absorbed per m 2 is 8×10^{-9} Wm $^{-2}$
 (4) the maximum kinetic energy of emitted photo electrons is 1.80 eV
13. A parallel beam of monochromatic light of wavelength 500 nm is incident normally on a perfectly absorbing surface. The power through any cross-section of the beam is 10 W.
- (1) the number of photons absorbed per second by the surface are 2.52×10^{19}
 (2) the number of photons absorbed per second by the surface are 4.76×10^{19}
 (3) the force exerted by the light beam on the surface is 3.33×10^{-8} N
 (4) the force exerted by the light beam on the surface is 1.5×10^{-8} N

Linked Comprehension Type

For Problems 1–3

Photoelectric threshold of silver is $\lambda = 3800 \text{ \AA}$. Ultraviolet light of $\lambda = 2600 \text{ \AA}$ is incident on silver surface. (Mass of the electron $9.11 \times 10^{-31} \text{ kg}$)

- Calculate the value of work function in eV.
(1) 1.77 (2) 3.27
(3) 5.69 (4) 2.32
- Calculate the maximum kinetic energy (in eV) of the emitted photoelectrons.
(1) 1.51 (2) 2.36
(3) 3.85 (4) 4.27
- Calculate the maximum velocity (in ms^{-1}) of the photoelectrons.
(1) 72.89×10^8 (2) 57.89×10^8
(3) 42.93×10^8 (4) 68.26×10^8

For Problems 4–6

A 100 W point source emits monochromatic light of wavelength $6000 \text{ \AA}$.

- Calculate the total number of photons emitted by the source per second.
(1) 5×10^{20} (2) 8×10^{20}
(3) 6×10^{21} (4) 3×10^{20}
- Calculate the photon flux (in SI unit) at a distance of 5 m from the source. Given $h = 6.6 \times 10^{-34} \text{ Js}$ and $c = 3 \times 10^8 \text{ m s}^{-1}$.
(1) 10^{15} (2) 10^{18}
(3) 10^{20} (4) 10^{22}
- 1.5 mW of $4000 \text{ \AA}$ light is directed at a photoelectric cell. If 0.10 per cent of the incident photons produce photoelectrons, find current in the cell. [Given $h = 6.6 \times 10^{-34} \text{ Js}$, $c = 3 \times 10^8 \text{ m s}^{-1}$ and $e = 1.6 \times 10^{-19} \text{ C}$]
(1) $0.59 \text{ \mu A}$ (2) $1.16 \text{ \mu A}$
(3) $0.48 \text{ \mu A}$ (4) $0.79 \text{ \mu A}$

For Problems 7–8

A metallic surface is illuminated alternatively with lights of wavelengths $3000 \text{ \AA}$ and $6000 \text{ \AA}$. It is observed that the maximum speeds of the photoelectrons under these illuminations are in the ratio 3:1.

- The work function of the metal is
(1) 1.45 eV (2) 2.26 eV
(3) 1.23 eV (4) 3.4 eV
- What will be the maximum speed of the photoelectrons so emitted?
(1) $9 \times 10^5 \text{ m s}^{-1}$ (2) $9 \times 10^7 \text{ m s}^{-1}$
(3) $3.6 \times 10^5 \text{ m s}^{-1}$ (4) $4.5 \times 10^7 \text{ m s}^{-1}$

For Problems 9–10

A helium–neon laser has a power output of 1 mW of light of wavelength 632.8 nm .

- Calculate the energy of each photon in electron volt.
(1) 2.5 (2) 1.96
(3) 0.53 (4) 3.3
- Determine the number of photons emitted by the laser each second.
(1) 3.18×10^{15} (2) 4.5×10^{16}
(3) 1.2×10^{15} (4) 2.9×10^{17}

For Problems 11–13

Photoelectrons are ejected from a surface when light of wavelength $\lambda_1 = 550 \text{ nm}$ is incident on it. The stopping potential for such electrons is $V_{s1} = 0.19 \text{ V}$. Suppose that radiation of wavelength $\lambda_2 = 190 \text{ nm}$ is incident on the surface.

- Calculate the stopping potential V_{s2} .
(1) 4.47 (2) 3.16
(3) 2.76 (4) 5.28
- Calculate the work function of the surface.
(1) 3.75 (2) 2.07
(3) 4.20 (4) 3.60
- Calculate the threshold frequency for the surface.
(1) $500 \times 10^{12} \text{ Hz}$ (2) $480 \times 10^{13} \text{ Hz}$
(3) $520 \times 10^{11} \text{ Hz}$ (4) $460 \times 10^{13} \text{ Hz}$

For Problems 14–15

In a photoelectric effect experiment, a metallic surface of work function 2.2 eV is illuminated with a light of wavelength 400 nm . Assume that an electron makes two collisions before being emitted and in each collision 10% additional energy is lost.

- Find the kinetic energy of this electron as it comes out of the metal.
(1) 0.46 eV (2) 0.31 eV
(3) 0.23 eV (4) none of these
- After how many collisions will the electron be unable to come out of the metal?
(1) 2 (2) 6
(3) 4 (4) 8

For Problems 16–18

In a photoelectric setup, a point source of light of power $3.2 \times 10^{-3} \text{ W}$ emits monoenergetic photons of energy 5.0 eV. The source is located at a distance of 0.8 m from the center of a stationary metallic sphere of work function 3.0 eV and of radius $8.0 \times 10^{-3} \text{ m}$. The efficiency of photoelectron emission is 1 for every 10^6 incident photons. Assume that the sphere is isolated and initially neutral and that photoelectrons are instantaneously swept away after emission.

- Calculate the number of photoelectrons emitted per second.
(1) 10^3 (2) 10^4
(3) 5×10^4 (4) 10^5
- It is observed that photoelectron emission stops at a certain time t after the light source is switched on. It is due to the retarding potential developed in the metallic sphere due to left over positive charges. The stopping potential (V) can be represented as

- (1) $2(K_{\max}/e)$ (2) $(K_{\max}/e)$
 (3) $(K_{\max}/3e)$ (4) $(K_{\max}/2e)$

18. Evaluate time t (in minutes).

- (1) 1.85 (2) 2.36
 (3) 2.75 (4) 0.78

For Problems 19–20

The incident intensity on a horizontal surface at sea level from the sun is about 1 kW m^{-2} .

19. Assuming that 50 per cent of this intensity is reflected and 50 per cent is absorbed, determine the radiation pressure on this horizontal surface (in pascals).

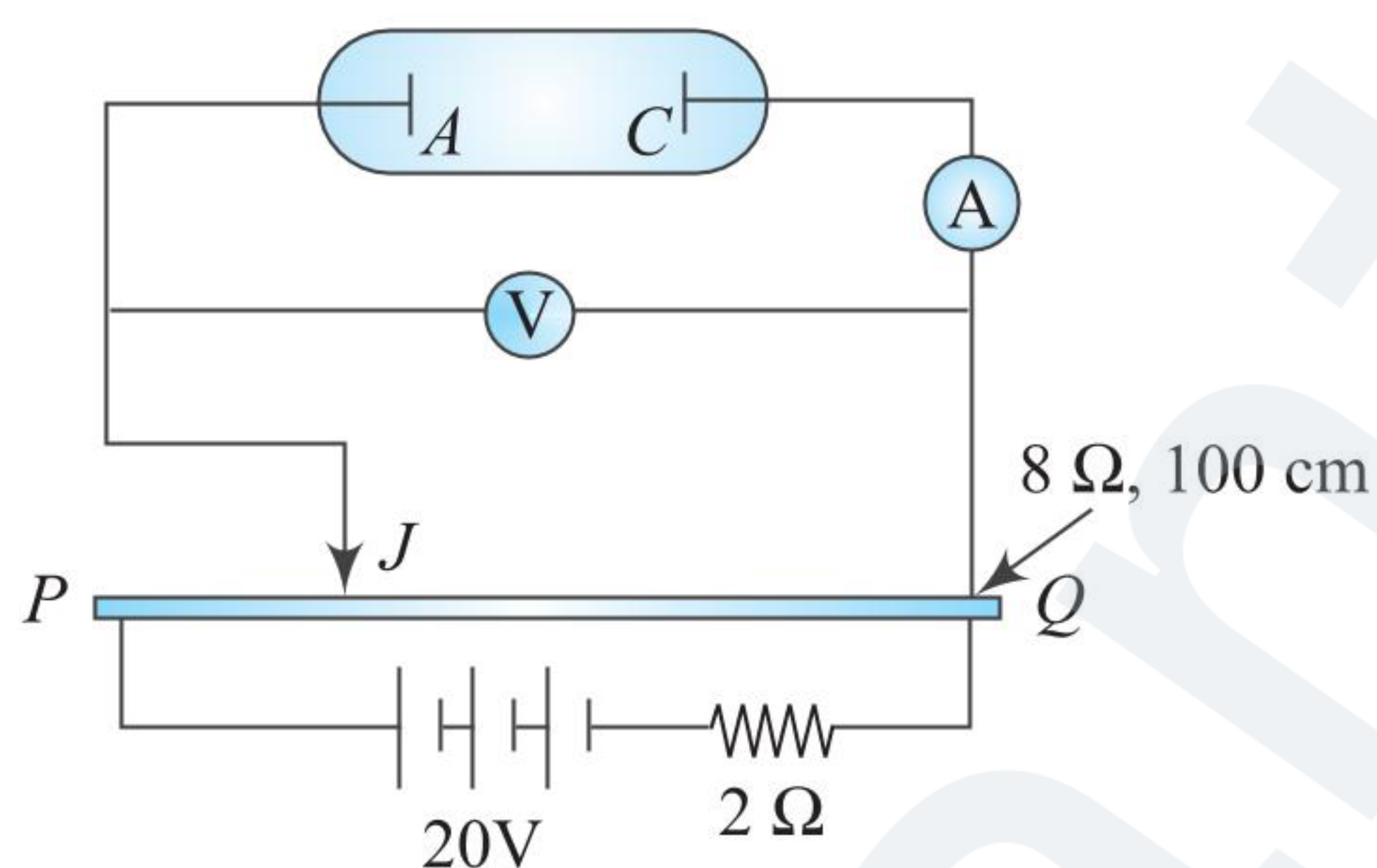
- (1) 8.2×10^{-2} (2) 5×10^{-6}
 (3) 3×10^{-5} (4) 6×10^{-5}

20. Find the ratio of this pressure to atmospheric pressure p_0 (about $1 \times 10^5 \text{ Pa}$) at sea level.

- (1) 5×10^{-11} (2) 4×10^{-8}
 (3) 6×10^{-12} (4) 8×10^{-11}

For Problems 21–25

An experimental setup of verification of photoelectric effect is shown in figure. The voltage across the electrodes is measured with the help of an ideal voltmeter, and which can be varied by moving jockey J on the potentiometer wire. The battery used in potentiometer circuit is of 20 V and its internal resistance is 2Ω . The resistance of 100 cm long potentiometer wire is 8Ω .



The photocurrent is measured with the help of an ideal ammeter. Two plates of potassium oxide of area 50 cm^2 at separation 0.5 mm are used in the vacuum tube. Photocurrent in the circuit is very small, so we can treat the potentiometer circuit as an independent circuit.

The wavelength of various colors is as follows:

Light	Violet	Blue	Green	Yellow	Orange	Red
λ in $\text{\AA}$	4000 to 5000	4500 to 5000	5000 to 5500	5500 to 6000	6000 to 6500	6500 to 7000

21. Calculate the number of electrons that appear on the surface of the cathode plate, when the jockey is connected at the end P of the potentiometer wire. Assume that no radiation is falling on the plates.

- (1) 8.85×10^6 (2) 11.0625×10^9
 (3) 8.85×10^9 (4) 0

22. When radiation falls on the cathode plate, a current of $2 \mu\text{A}$ is recorded in the ammeter. Assuming that the vacuum tube setup follows Ohm's law, the equivalent resistance of vacuum tube operating in this case when jockey is at end P is

- (1) $8 \times 10^8 \Omega$ (2) $16 \times 10^6 \Omega$
 (3) $8 \times 10^6 \Omega$ (4) $10 \times 10^6 \Omega$

23. It is found that ammeter current remains unchanged ($2 \mu\text{A}$) even when the jockey is moved from the end P to the middle point of the potentiometer wire. Assuming that all the incident photons eject electrons and the power of the light incident is $4 \times 10^{-6} \text{ W}$. Then, the color of the incident light is

- (1) green (2) violet
 (3) red (4) orange

24. Which of the following colors may not give photoelectric effect for this cathode?

- (1) green (2) violet
 (3) red (4) orange

25. When other light falls on the anode plate, the ammeter reading remains zero till jockey is moved from the end P to the middle point of the wire PQ . Thereafter, the deflection is recorded in the ammeter. The maximum kinetic energy of the emitted electron is

- (1) 16 eV (2) 8 eV
 (3) 4 eV (4) 10 eV

For Problems 26–28

Light having photon energy $h\nu$ is incident on a metallic plate having work function ϕ to eject the electrons. The most energetic electrons are then allowed to enter in a region of uniform magnetic field B as shown in figure. The electrons are projected in X - Z plane making an angle θ with X -axis and magnetic field is $\vec{B} = B_0 \hat{i}$ along X -axis.

Maximum pitch of the helix described by an electron is found to be p . Take mass of electron as m and charge as q .

Based on the above information, answer the following questions:

26. The correct relation between p and B_0 is

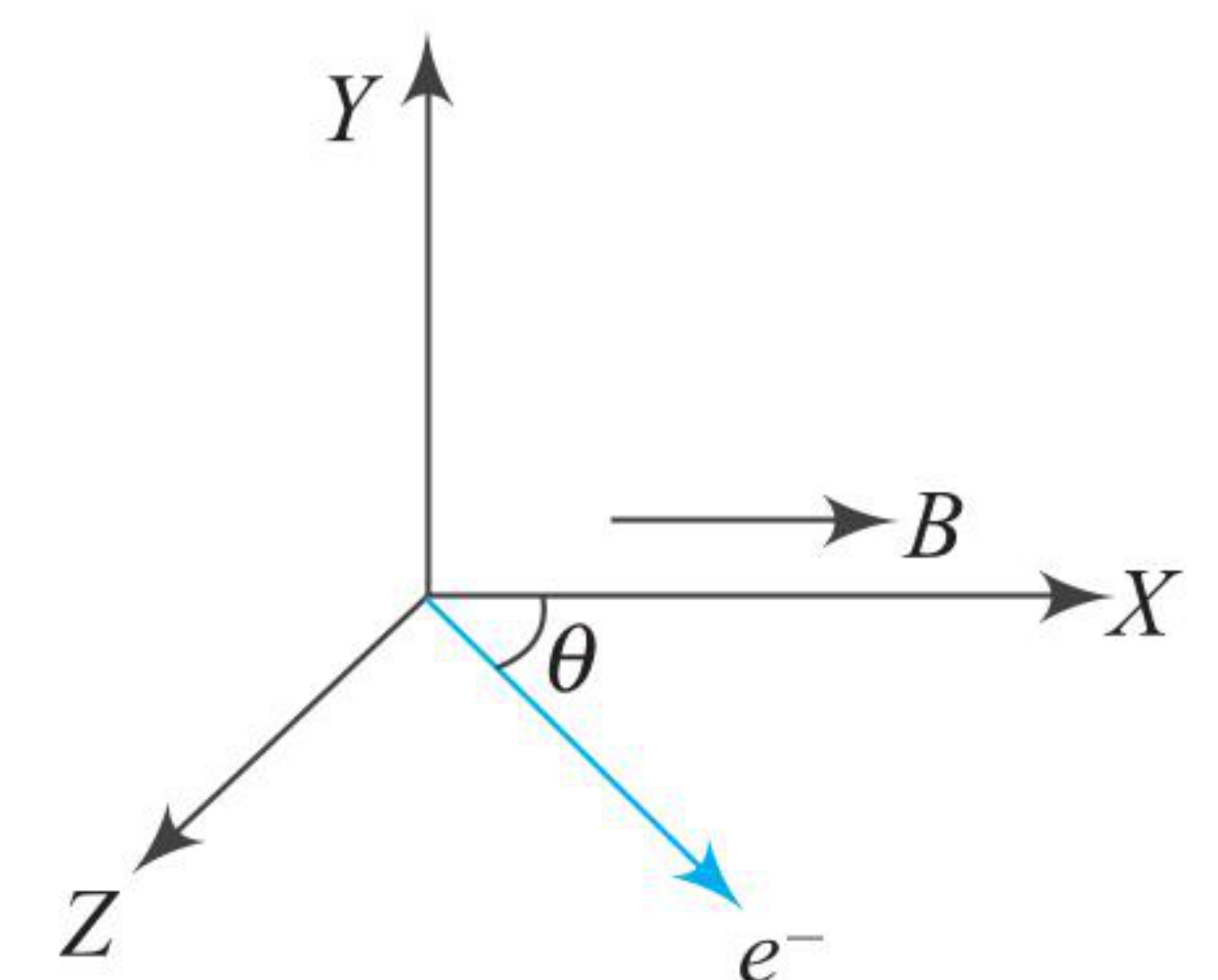
(1) $qpB_0 = 2\pi \cos \theta \sqrt{2(h\nu - \phi)m}$

(2) $qpB_0 = 2\pi \cos \theta \sqrt{\frac{2(h\nu - \phi)}{m}}$

(3) $pqB_0 = 2\pi \sqrt{2(h\nu - \phi)m}$

(4) $p = \frac{2\pi m}{qB_0} \times \sqrt{h\nu - \phi}$

27. Considering the instant of crossing origin at $t = 0$, the Z -coordinate of the location of electron as a function of time is



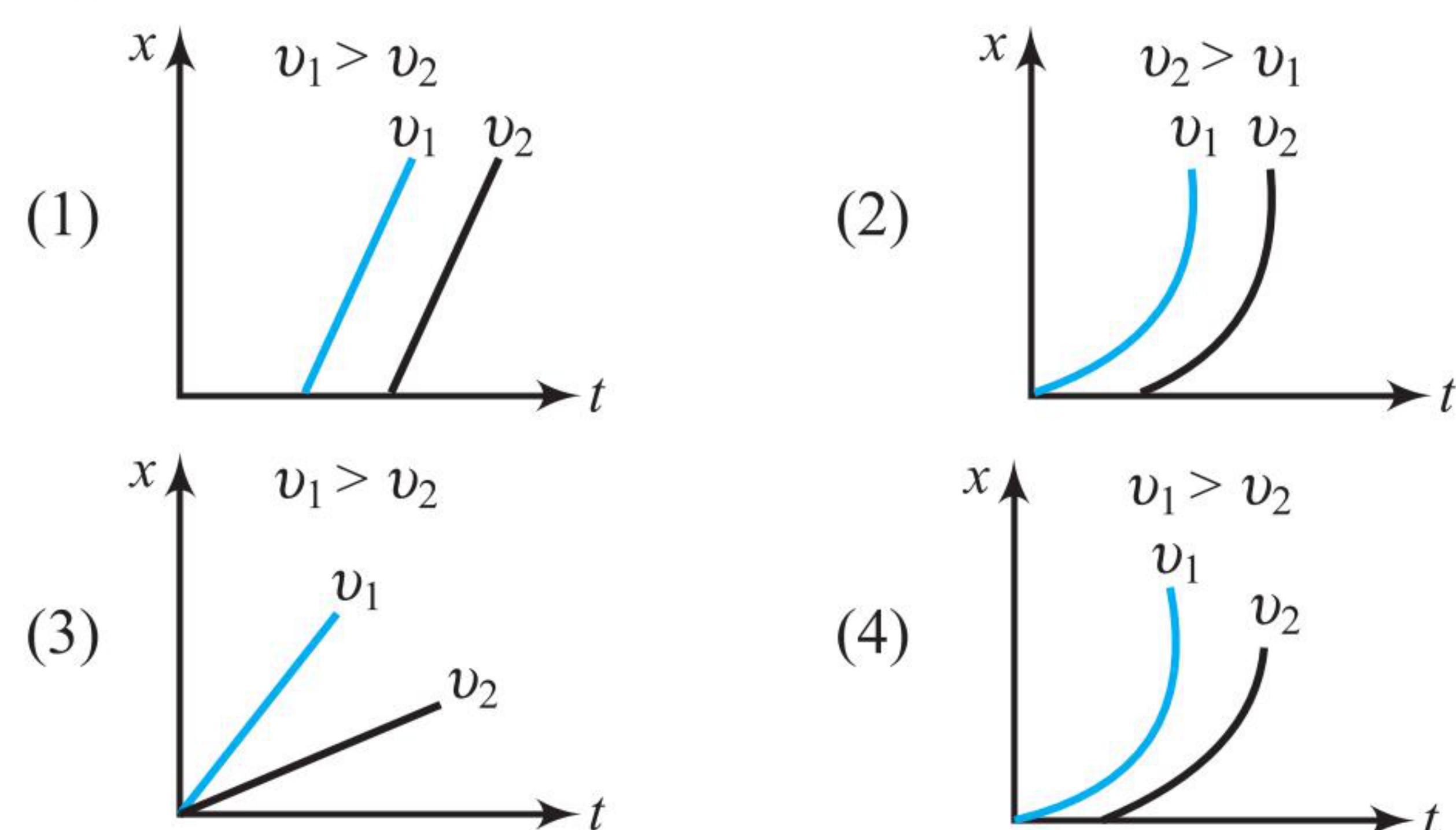
$$(1) -\frac{\sqrt{2m(h\nu - \phi)}}{qB_0} \times \sin \theta \left[1 - \cos \left(\frac{qB_0 t}{m} \right) \right]$$

$$(2) \frac{\sqrt{2m(h\nu - \phi)}}{qB_0} \times \sin \theta \times \sin \left[\frac{qB_0 t}{m} \right]$$

$$(3) \frac{-\sqrt{2m(h\nu - \phi)}}{qB_0} \times \sin \theta \times \sin \left[\frac{qB_0 t}{m} \right]$$

$$(4) \frac{\sqrt{2m(h\nu - \phi)}}{qB_0} \times \sin \left(\frac{qB_0 t}{m} \right)$$

28. The plot between X -coordinate of the location of electron as a function of time for different frequencies ν of the incident light is



For Problems 29–31

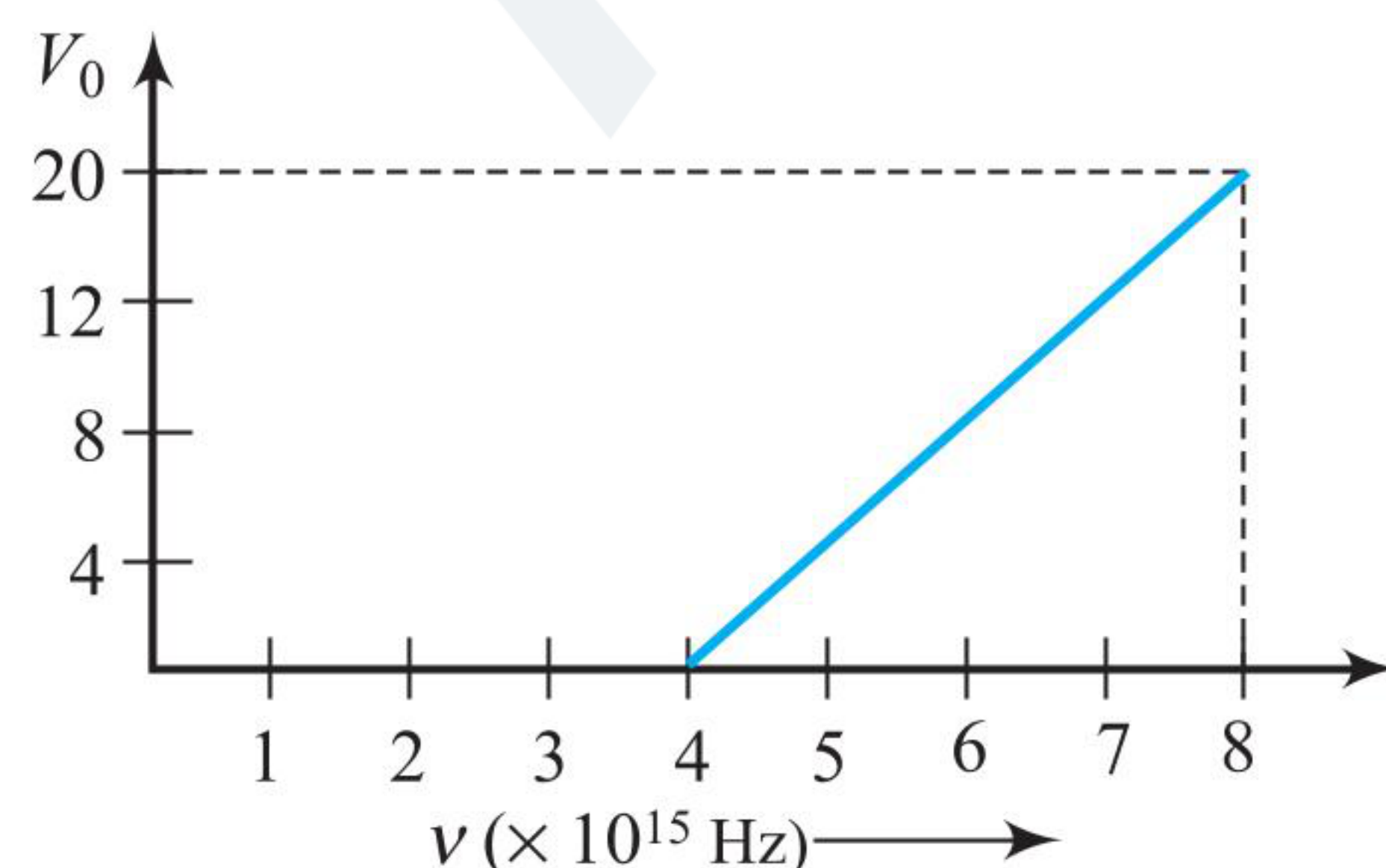
When a high frequency electromagnetic radiation is incident on a metallic surface, electrons are emitted from the surface. Energy of emitted photoelectrons depends only on the frequency of incident electromagnetic radiation and the number of emitted electrons depends only on the intensity of incident light.

Einstein's photoelectric equation [$K_{\max} = h\nu - \phi$] correctly explains the PE, where ν = frequency of incident light and ϕ = work function.

29. Light of wavelength 3300 Å is incident on two metals A and B , whose work functions are 4 eV and 2 eV, respectively. Then

- (1) A will emit photoelectrons but B will not
- (2) B will emit photoelectrons, but A will not
- (3) both A and B will not emit photoelectrons
- (4) neither A nor B will emit photoelectrons

30. For photoelectric effect in a metal, the graph of the stopping potential V_0 (in volt) versus frequency ν (in hertz) of the incident radiation is shown in figure. The work function of the metal (in eV) is



- (1) 12.5
- (2) 14.5
- (3) 16.5
- (4) 18.5

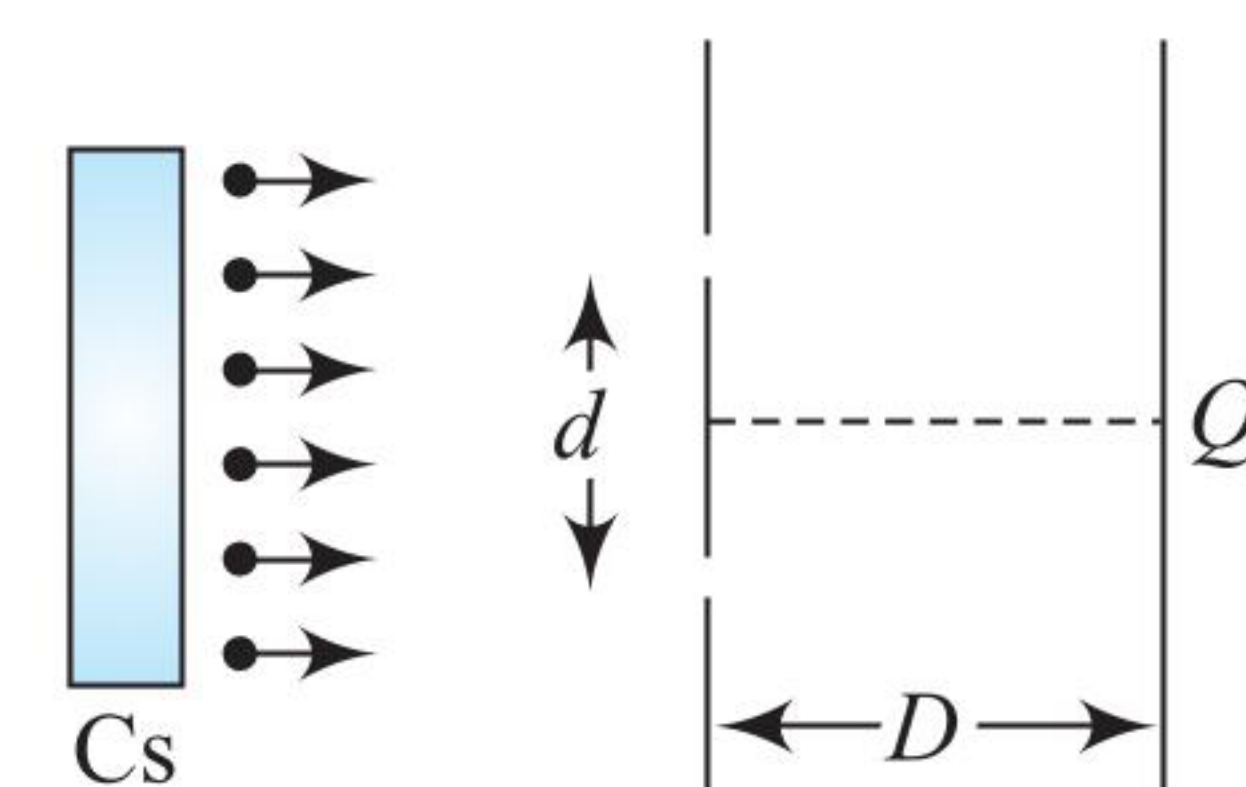
31. The slope of the graph shown in figure [here h is the Planck's constant and e is the amount of charge on an electron] is

- (1) $\frac{h}{e}$
- (2) eh
- (3) h
- (4) $\frac{e}{h}$

For Problems 32–34

A Cs plate is irradiated with a light of wavelength $\lambda = hc/\phi$, ϕ being the work function of the plate, h Planck's constant, and c the velocity of light in vacuum.

Assume all the photoelectrons are moving perpendicular to the plate toward a YDSE setup when accelerated through a potential difference V . Take charge on a proton = e and mass of an electron = m .



32. The fringe width due to the electron beam is

- (1) $\lambda D/d$
- (2) $\lambda D/2d$
- (3) $hD/(d\sqrt{2emV})$
- (4) none of these

33. If the wavelength of light used in photoemission is less than λ , then the fringe width will

- (1) increase
- (2) decrease
- (3) remain same
- (4) cannot be decided

34. Instead of moving perpendicular to the plate, if the electrons were moving randomly, then the central maximum would shift

- (1) upward
- (2) downward
- (3) no shift
- (4) no fringes will be formed

For Problems 35–37

A pushed dye laser emits light of wavelength 585 nm. Because this wavelength is strongly absorbed by the haemoglobin in the blood, the method is especially effective for removing various types of blemishes due to blood. To get a reasonable estimate of the power required for such laser surgery, we can model the blood as having the same specific heat and heat of vaporization as water. [$S = 4.2 \times 10^3 \text{ J (kg K)}^{-1}$, $L = 2.25 \times 10^6 \text{ J kg}^{-1}$]

35. Suppose that each pulse must remove 2 μg of blood by evaporating it starting at 30°C. The energy that each pulse must deliver to the blemish is nearly

- (1) 5.1 J
- (2) 5.1 mJ
- (3) 5.1 μJ
- (4) 5.1 kJ

36. The power output of the laser must be

- (1) 5.5 W (2) 11 W
(3) 16.5 W (4) 22 W

37. The number of photons that each pulse delivers to the blemish is

- (1) 1.5×10^{16} (2) 1.5×10^8
(3) 3×10^{16} (4) 3×10^8

For Problems 38–39

Some phenomenon of light can be explained by wave theory and some other phenomenon show the particle nature of light. de-Broglie proposed a hypothesis based on the dual nature of light. According to de-Broglie each moving particle has a wave associated with it. This wave is known as de-Broglie wave or

matter wave and its wavelength is given by $\lambda = \frac{h}{p}$. Actually

this expression correlate wave and particle nature. Wavelength (λ) represents the wave nature and momentum (p) represents the particle nature.

38. Proton, deuteron and α particles are accelerated through the same potential difference, then the ratio of their wavelength is

- (1) $1:\sqrt{2}:1$ (2) $1:1:1$
(3) $1:2:2\sqrt{2}$ (4) $2\sqrt{2}:2:1$

39. When electron accelerated through the 150 volt potential difference. The wavelength associated with it

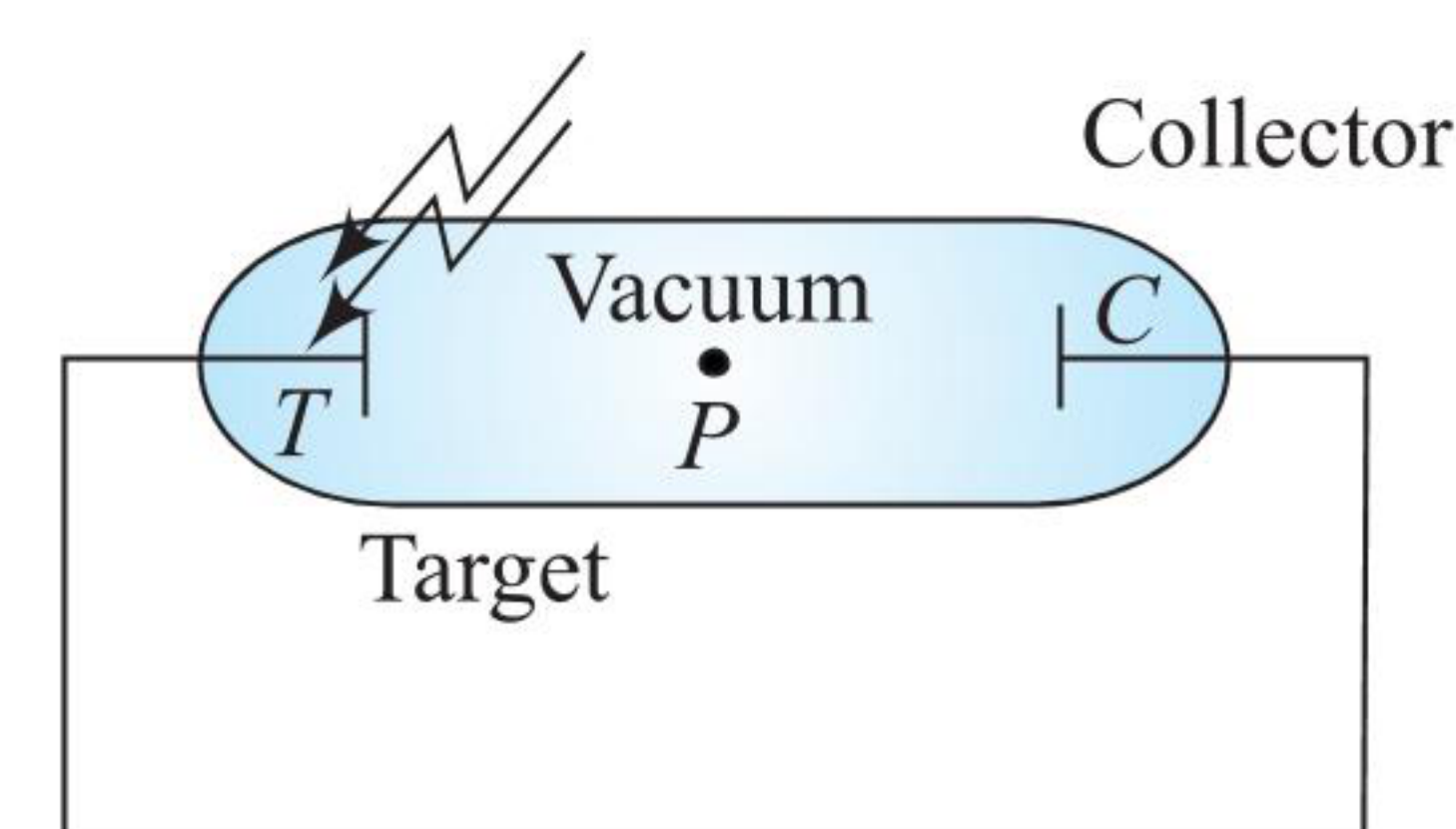
- (1) 1 Å (2) 2 Å
(3) $1/2$ Å (4) 4 Å

Matrix Match Type

1. With respect to photoelectric effect experiment, match the entries of Column I with the entries of Column II.

Column I	Column II
i. If f (frequency) is increased keeping I (intensity) and ϕ (work function) constant	a. stopping potential increases
ii. If I is increased keeping f and ϕ constant	b. saturation photocurrent increases
iii. If the distance between anode and cathode increases	c. maximum KE of the photoelectrons increases
iv. If ϕ is decreased keeping f and I constant.	d. stopping potential remains the same

2. In a photoelectric experimental arrangement, light of frequency f is incident on a metal target whose work function is $\phi = hf/3$ as shown. In Column I, KE of photoelectron is mentioned at various locations/instants and in Column II, the corresponding values. Match the entries of Column I with the entries of Column II.

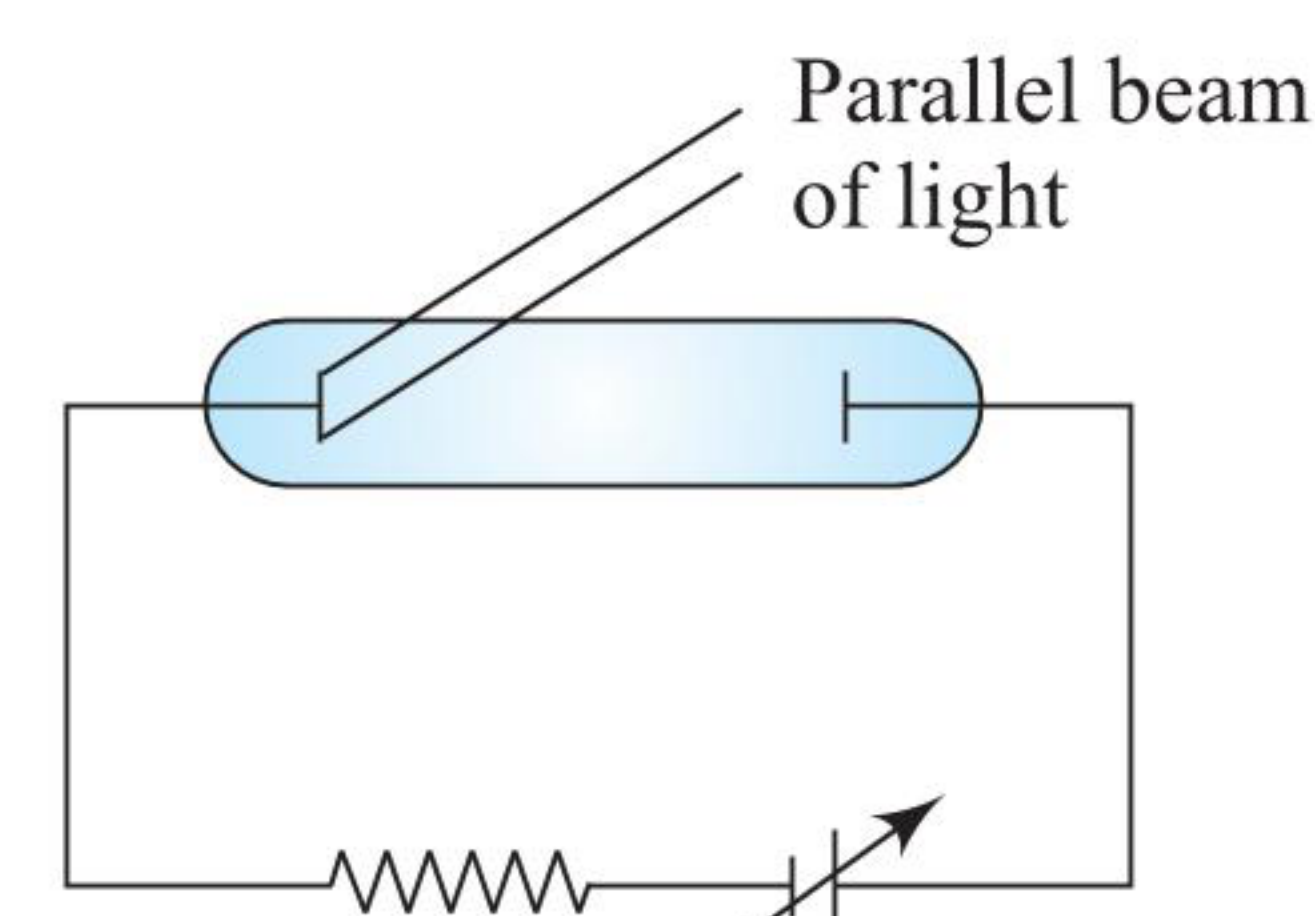


Column I	Column II
i. Maximum KE of photoelectrons just after emission from target	a. zero
ii. KE of photoelectrons just after emission from target	b. $hf/3$
iii. KE of photoelectron when they are halfway between the target and collector	c. $hf/2$
iv. KE of photoelectrons as they reach the collector	d. $2hf/3$

3. Related to photoelectric effect, in Column I, some physical quantities change while in Column II effects of these changes are given. Match the entries of Column I with the entries of Column II.

Column I	Column II
i. Intensity of incident light changes	a. $K_{\max}$ of emitted photo-electrons changes
ii. Frequency of incident light changes	b. Stopping potential changes
iii. Target material changes	c. Saturation current changes
iv. Potential difference between the emitter and collector changes	d. Time delay in emission of photoelectrons changes

4. In the shown experimental setup to study photoelectric effect, two conducting electrodes are enclosed in an evacuated glass-tube as shown. A parallel beam of monochromatic light falls on photosensitive electrode. The emf of battery shown is high enough such that all photoelectrons ejected from left electrode will reach the right electrode. Under initial conditions, photoelectrons are emitted. As changes are made in each situation of Column I. Match the statement in Column I with results in Column II.

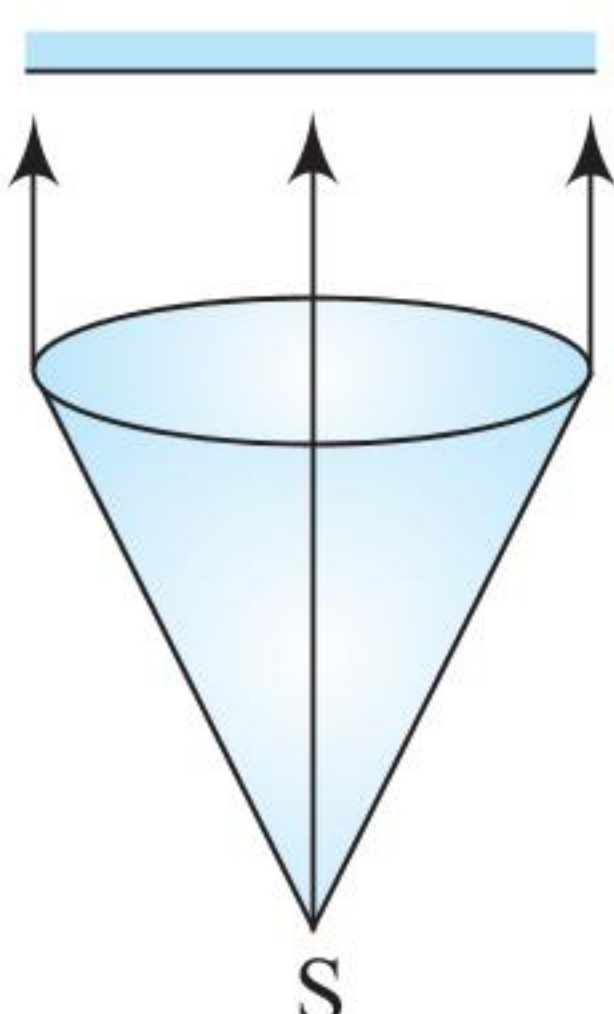
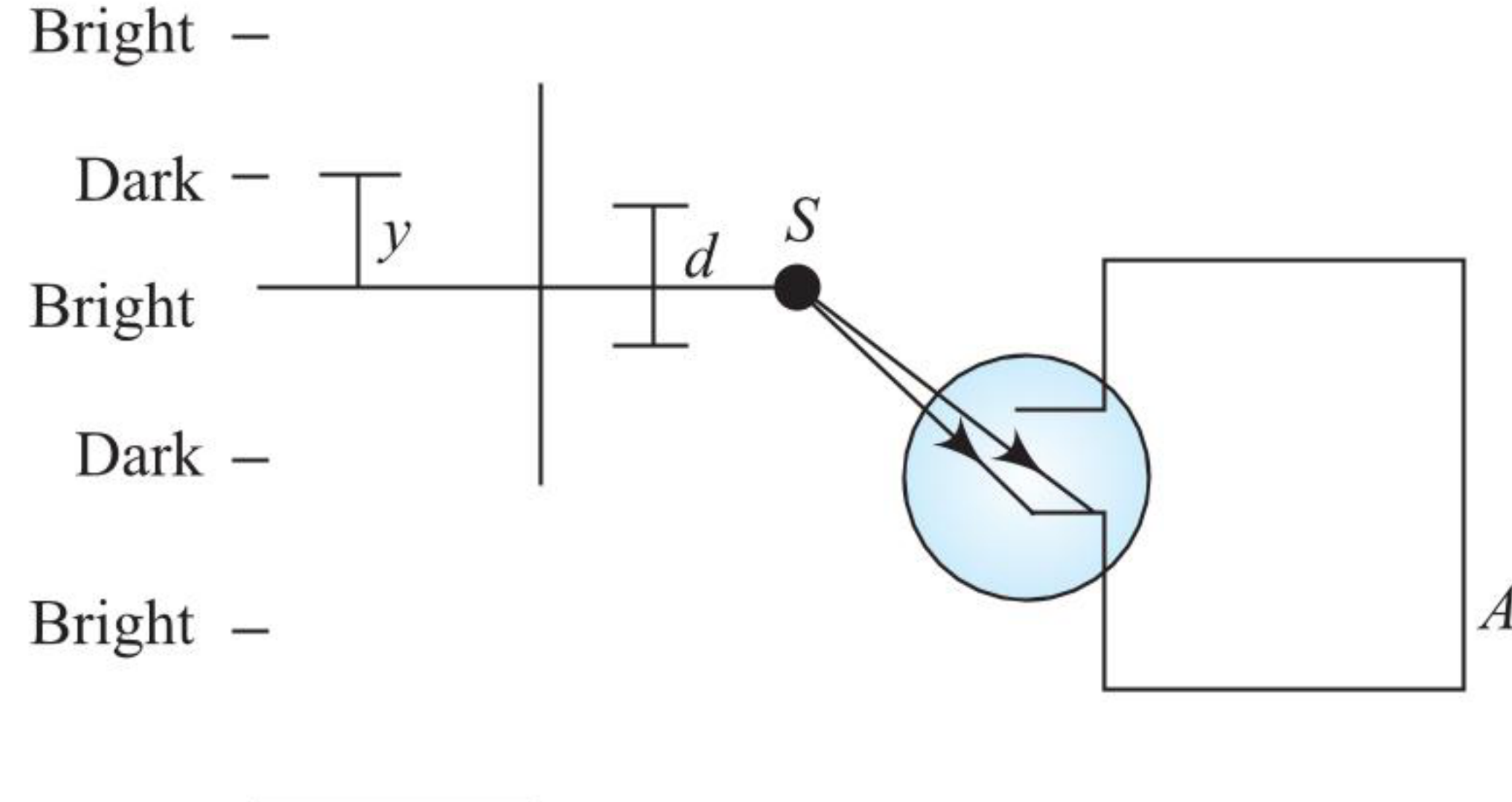
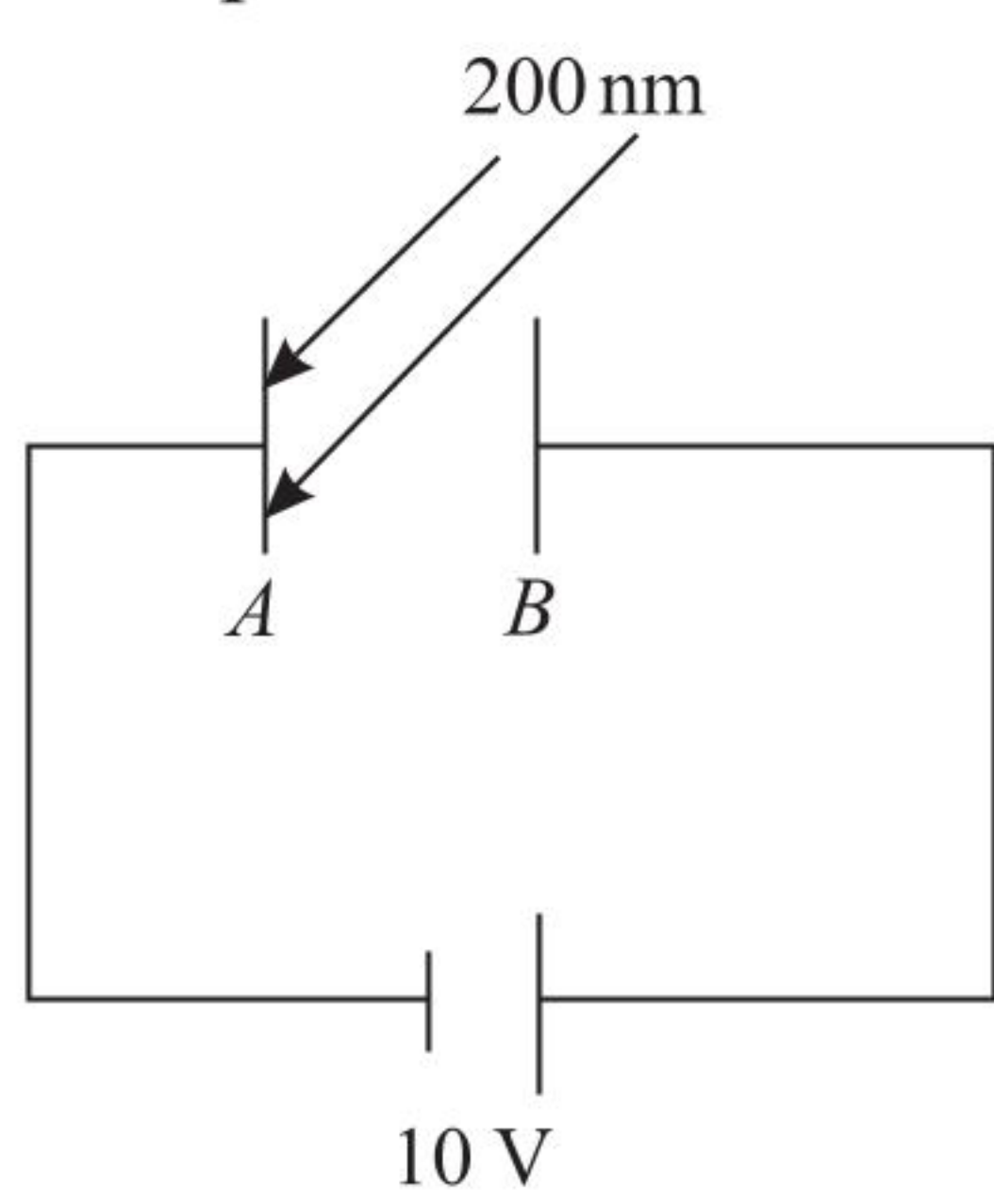


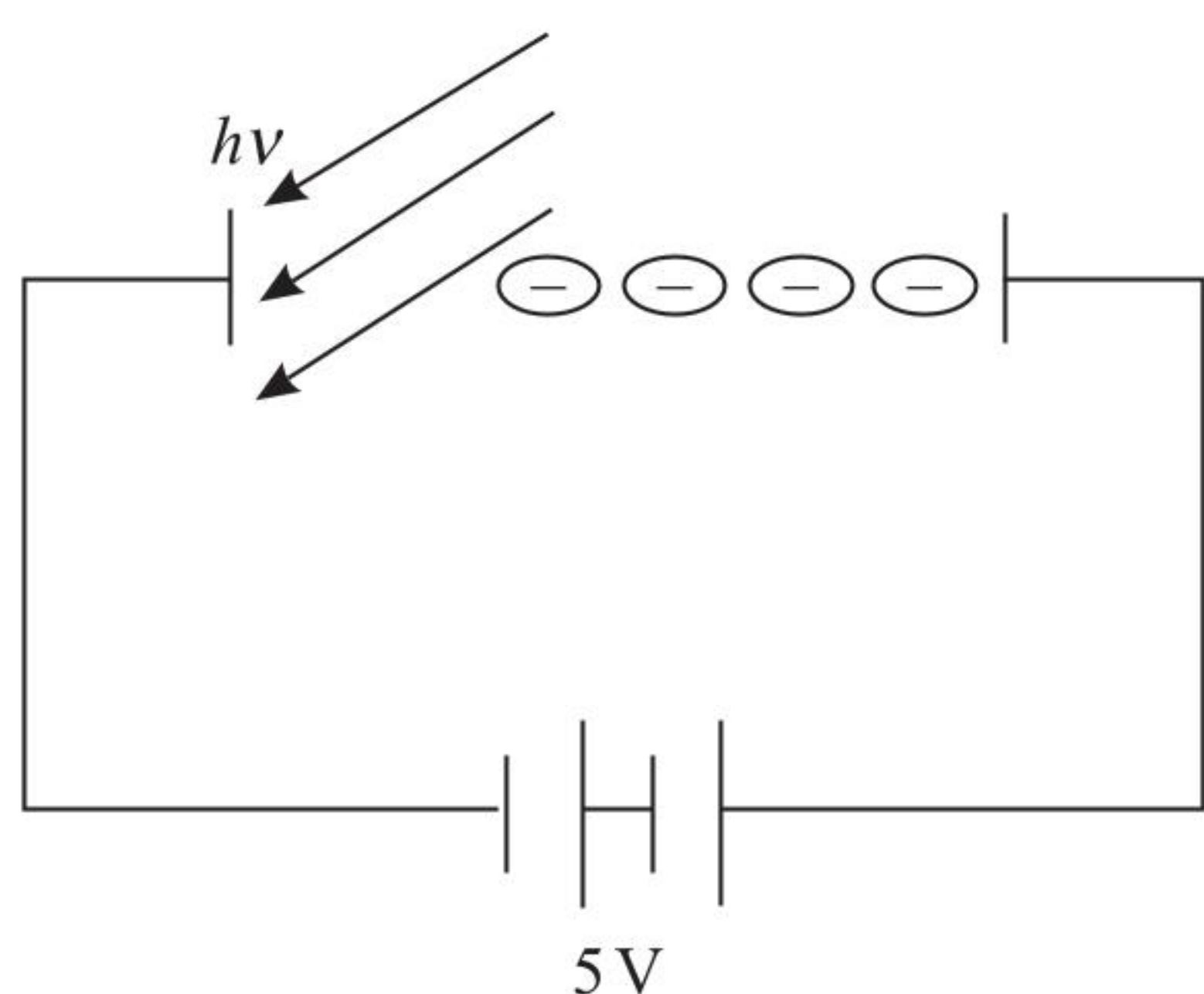
Column I	Column II
i. If frequency of incident light is increased keeping its intensity constant	a. magnitude of stopping potential will increase
ii. If frequency of incident light is increased and its intensity is decreased	b. current through the circuit may stop
iii. If work function of photon sensitive electrode is increased	c. maximum kinetic energy of ejected photoelectrons will increase
iv. If intensity of incident light is increased keeping its frequency constant	d. saturation current will increase

5. In Column I, the nature of light is given and in Column II the information about the photons are mentioned. Match the entries of Column I with the entries of Column II.

Column I	Column II
i. A bichromatic light source	a. Few photons have same energy and momenta
ii. A point source of white light emitting light uniformly in all directions	b. Few photons have different energy and different momenta
iii. A point source of monochromatic light emitting light uniformly in all directions	c. Few photons have same energy and different momenta
iv. Laser light source	d. Few photons have different energy and same momenta

Numerical Value Type

- The radius of an α -particle moving in a circle in a constant magnetic field is half of the radius of an electron moving in circular path in the same field. The de-Broglie wavelength of α -particle is n times that of the electron. Find n (an integer).
- The de-Broglie wavelength of an electron moving with a velocity of $1.5 \times 10^8 \text{ m s}^{-1}$ is equal to that of a photon. Find the ratio of the kinetic energy of the photon to that of the electron.
- An element of atomic number 9 emits K_α X-ray of wavelength λ . Find the atomic number of the element which emits K_α X-ray of wavelength 4λ .
- A monochromatic source of light operating at 200 W emits 4×10^{20} photons per second. Find the wavelength of the light (in $\times 10^{-7} \text{ m}$).
- A parallel beam of monochromatic light of wavelength 663 nm is incident on a totally reflecting plane mirror. The angle of incidence is 60° and the number of photons striking the mirror per second is 1.0×10^{19} . Calculate the force exerted by light beam on the mirror (in 10^{-8} N).
- A totally reflecting, small plane mirror placed horizontally faces a parallel beam of light as shown in the figure. The mass of the mirror is 20 g. Assume that there is no absorption in the lens and that 30% of the light emitted by the source goes through the lens. Find the power (in $\times 10^8 \text{ W}$) of the source needed to support the weight of the mirror. Take $g = 10 \text{ m s}^{-2}$.

- A silver ball of radius 4.8 cm is suspended by a thread in a vacuum chamber. Ultraviolet light of wavelength 200 nm is incident on the ball for some time during which a total light energy of $1.0 \times 10^{-7} \text{ J}$ falls on the surface. Assuming that on the average, one photon out of ten thousand photons is able to eject a photoelectron, find the electric potential (in $\times 10^{-1} \text{ V}$) at the surface of the ball assuming zero potential at infinity.
- In the arrangement shown in figure, $y = 1.0 \text{ mm}$, $d = 0.24 \text{ mm}$ and $D = 1.2 \text{ m}$. The work function of the material of the emitter is 2.2 eV. Find the stopping potential V needed to stop the photocurrent (in $\times 10^{-1} \text{ V}$).

- An electron of mass m when accelerated through a potential difference V , has de Broglie wavelength λ . The de-Broglie wavelength associated with a particle of mass $m/4$ and charge e accelerated through the same potential difference is $n\lambda$. Then find the value of n .
- In figure, electromagnetic radiations of wavelength 200 nm are incident on a metallic plate A. The photoelectrons are accelerated by a potential difference of 10 V. These electrons strike another metal plate B from which electromagnetic radiations are emitted. The minimum wavelength of emitted photons is 100 nm. If the work function of metal A is found to be $(x \times 10^{-1}) \text{ eV}$, then find the value of x . (Given $hc = 1240 \text{ eV nm}$)

- Photons of energy 5 eV are incident on the cathode. Electrons reaching the anode have kinetic energies varying from 6 eV to 8 eV Find the work function of the metal (in eV).



12. A plane light wave of intensity $I = 0.20 \text{ W cm}^{-2}$ falls on a plane mirror surface with reflection coefficient $\rho = 0.8$. The angle of incidence is 45° . In terms of corpuscular theory, find the magnitude of the normal pressure exerted on that surface (in N cm^{-2}).
13. Light of wavelength 180 nm ejects photoelectrons from a plate of a metal whose work function is 2 eV . If a uniform

magnetic field of $5 \times 10^{-5} \text{ T}$ is applied parallel to the plate, what would be the radius of the path (in m) followed by electrons ejected normally from the plate with maximum energy.

14. Calculate de-Broglie wavelength (in $\text{\AA}$) for an average helium atom in a furnace at 400 K . Given mass of helium = 4.002 amu .
15. What amount of energy (in eV) should be added to an electron to reduce its de-Broglie wavelength from 100 to 50 pm ?
16. A cylindrical rod of some laser material $5 \times 10^{-2} \text{ m}$ long and 10^{-2} m in diameter contains 2×10^{25} ions per m^3 . If on excitation all the ions are in the upper energy level and de-excite simultaneously emitting photons in the same direction, calculate the maximum energy contained in a pulse of radiation of wavelength $6.6 \times 10^{-7} \text{ m}$. If the pulse lasts for 10^{-7} s , calculate the average power (in W) of the laser during the pulse.

Archives

JEE ADVANCED

Single Correct Answer Type

1. A pulse of light of duration 100 ns is absorbed completely by a small object initially at rest. Power of the pulse is 30 mW and the speed of light is $3 \times 10^8 \text{ m/s}$. The final momentum of the object is

- (1) $0.3 \times 10^{-17} \text{ kg ms}^{-1}$ (2) $1.0 \times 10^{-17} \text{ kg ms}^{-1}$
(3) $0.2 \times 10^{-17} \text{ kg ms}^{-1}$ (4) $9.0 \times 10^{-17} \text{ kg ms}^{-1}$

(JEE Advanced 2013)

2. A metal surface is illuminated by light of two different wavelengths 248 nm and 310 nm . The maximum speeds of the photoelectrons corresponding to these wavelengths are u_1 and u_2 , respectively. If the ratio $u_1 : u_2 = 2 : 1$ and $hc = 1240 \text{ eV nm}$, the work function of the metal is nearly

- (1) 3.7 eV (2) 3.2 eV
(3) 2.8 eV (4) 2.5 eV

(JEE Advanced 2014)

3. In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured by applying a stopping potential. The relevant data for the wavelength (λ) of incident light and the corresponding stopping potential (V_0) are given below:

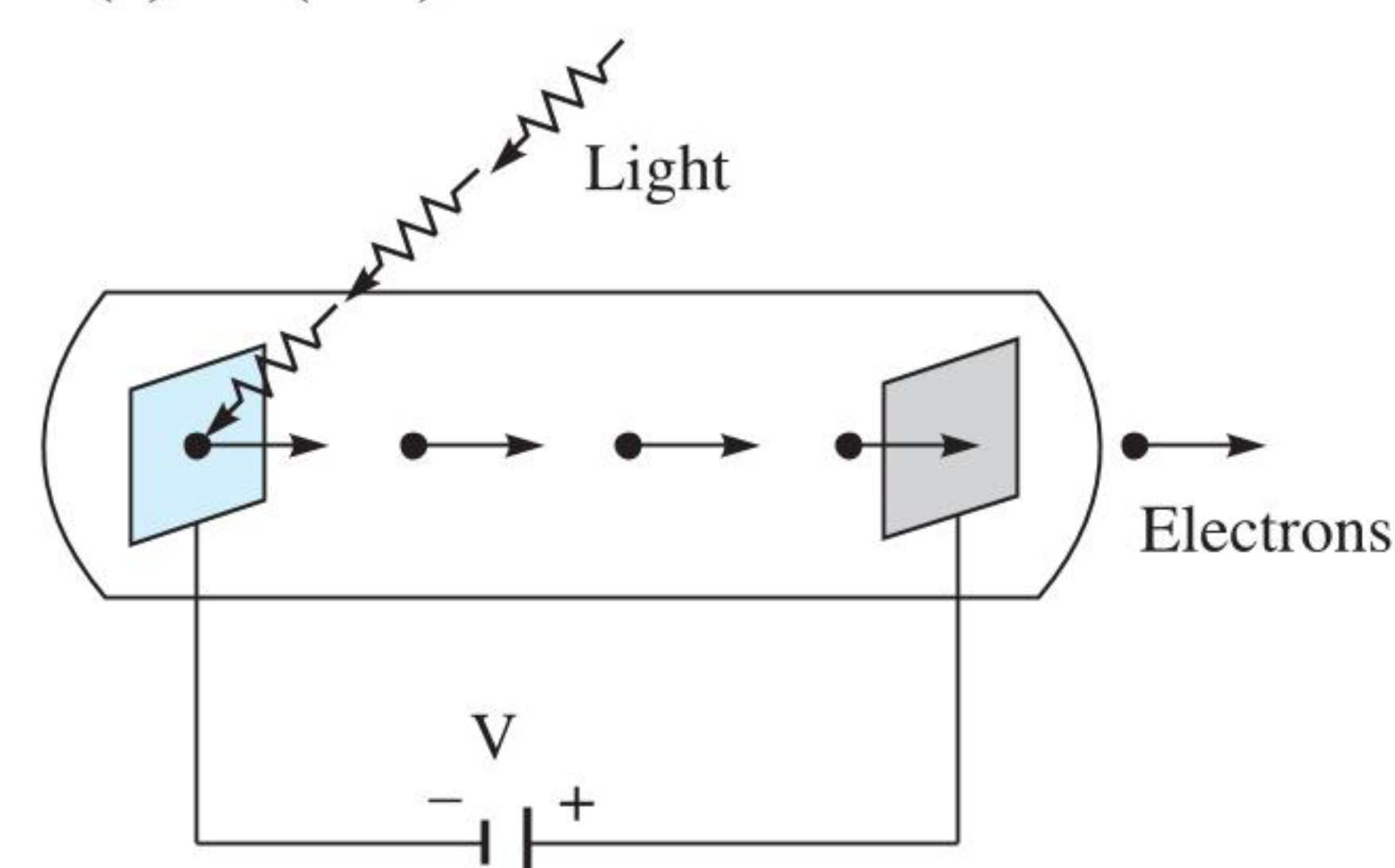
$\lambda (\mu\text{m})$	$V_0 (\text{Volt})$
0.3	2.0
0.4	1.0
0.5	0.4

Given that $c = 3 \times 10^8 \text{ ms}^{-1}$ and $e = 1.6 \times 10^{-19} \text{ C}$, Planck's constant (in units of J s) found from such an experiment is

- (1) 6.0×10^{-34} (2) 6.4×10^{-34}
(3) 6.6×10^{-34} (4) 6.8×10^{-34}

(JEE Advanced 2016)

4. Light of wavelength λ_{ph} falls on a cathode plate inside a vacuum tube as shown in the figure. The work function of the cathode surface is ϕ and the anode is a wire mesh of conducting material kept at a distance d from the cathode. A potential difference V is maintained between the electrodes. If the minimum de Broglie wavelength of the electrons passing through the anode is λ_e , which of the following statement(s) is (are) true?



- (1) λ_e increases at the same rate as λ_{ph} for $\lambda_{ph} < hc/\phi$.
(2) For large potential difference ($V \gg \phi/e$), λ_e is approximately halved if V is made four times
(3) λ_e is approximately halved, if d is doubled
(4) λ_e decreases with increase in ϕ and λ_{ph}

(JEE Advanced 2016)

5. A photoelectric material having work-function ϕ_0 is illuminated with light of wavelength λ ($\lambda < \frac{hc}{\phi_0}$).

The fastest photoelectron has a de-Broglie wavelength λ_d . A change in wavelength of the incident light by $\Delta\lambda$ results in a change $\Delta\lambda_d$ in λ_d . Then the ratio $\frac{\Delta\lambda_d}{\Delta\lambda}$ is proportional to

- (1) $\frac{\lambda_d^3}{\lambda^2}$ (2) $\frac{\lambda_d^3}{\lambda}$
(3) $\frac{\lambda_d^2}{\lambda^2}$ (4) $\frac{\lambda_d}{\lambda}$

(JEE Advanced 2017)

Linked Comprehension Type**For Problems 1–3**

When a particle is restricted to move along x -axis between $x = 0$ and $x = a$, where a is of nanometer dimension, its energy can take only certain specific values. The allowed energies of the particle moving in such a restricted region correspond to the formation of standing waves with nodes at its ends $x = 0$ and $x = a$. The wavelength of this standing wave is related to the linear momentum p of the particle according to the de Broglie relation. The energy of the particle of mass m is related to its linear momentum as $E = p^2/2m$. Thus, the energy of the particle can be denoted by a quantum number ' n ' taking values 1, 2, 3, ... ($n = 1$, called the ground state) corresponding to the number of loops in the standing wave.

Use the model described above to answer the following three questions for a particle moving along the line from $x = 0$ to $x = a$. Take $h = 6.6 \times 10^{-34}$ J s and $e = 1.6 \times 10^{-19}$ C.

(IIT-JEE 2009)

1. The allowed energy for the particle for a particular value of n is proportional to

(1) a^{-2} (2) $a^{-3/2}$
(3) a^{-1} (4) a^2

2. If the mass of the particle is $m = 1.0 \times 10^{-30}$ kg and $a = 6.6$ nm, the energy of the particle in its ground state is closest to

(1) 0.8 meV (2) 8 meV
(3) 80 meV (4) 800 meV

3. The speed of the particle that can take discrete values is proportional to

(1) $n^{-3/2}$ (2) n^{-1}
(3) $n^{1/2}$ (4) n

Numerical Value Type

1. An α -particle and a proton are accelerated from rest by a potential difference of 100 V. After this, their de Broglie wavelengths are λ_α and λ_p , respectively. The ratio λ_p/λ_α , to the nearest integer, is _____.

(IIT-JEE 2010)

2. A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in free-space. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is $A \times 10^Z$ (where $1 < A < 10$). The value of " Z " is _____.

(IIT-JEE 2011)

3. A proton is fired from very far away towards a nucleus with charge $Q = 120e$, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de-Broglie

wavelength (in units of fm) of the proton at its start is (take the proton mass, $m_p = (5/3) \times 10^{-27}$ kg; $h/e = 4.2 \times 10^{-15}$ Js/C;

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ m/F; } 1 \text{ fm} = 10^{-15} \text{ m}) \quad \textbf{(IIT-JEE 2012)}$$

4. The work functions of Silver and Sodium are 4.6 and 2.3 eV, respectively. The ratio of the slope of the stopping potential versus frequency plot for Silver to that of Sodium is _____.

(JEE Advanced 2013)

5. In a photoelectric experiment a parallel beam of monochromatic light with power of 200 W is incident on a perfectly absorbing cathode of work function 6.25 eV. The frequency of light is just above the threshold frequency so that the photoelectrons are emitted with negligible kinetic energy.

Assume that the photoelectron emission efficiency is 100%. A potential difference of 500 V is applied between the cathode and the anode. All the emitted electrons are incident normally on the anode and are absorbed. The anode experiences a force $F = n \times 10^{-4}$ N due to the impact of the electrons. The value of n is _____.

Mass of the electron $m_e = 9 \times 10^{-31}$ kg and

$$1.0 \text{ eV} = 1.6 \times 10^{-19} \text{ J.}$$

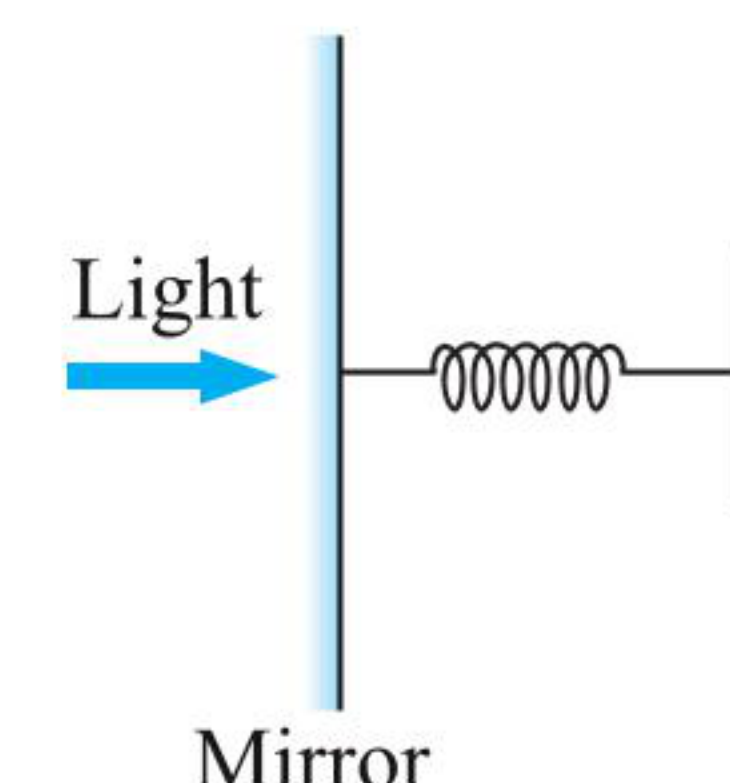
(JEE Advanced 2018)

6. A perfectly reflecting mirror of mass M mounted on a spring constitutes a spring mass system of angular frequency Ω

such that $\frac{4\pi M\Omega}{h} = 10^{24} \text{ m}^{-2}$, where h

is planck's constant. N photons of

wavelength $\lambda = 8\pi \times 10^{-6}$ m strike the mirror simultaneously at normal incidence such that the mirror gets displaced by 1 μm . If the value of N is $x \times 10^{12}$, then the value of x is _____ (consider the spring as massless).

(JEE Advanced 2019)

7. In a photoemission experiment, the maximum kinetic energies of photoelectrons from metals P , Q and R are E_P , E_Q and E_R , respectively, and they are related by $E_P = 2 E_Q = 2 E_R$. In this experiment, the same source of monochromatic light is used for metals P and Q while a different source of monochromatic light is used for the metal R . The work functions for metals P , Q and R are 4.0 eV, 4.5 eV and 5.5 eV, respectively. The energy of the incident photon used for metal R , in eV, is _____.

(JEE Advanced 2021)

Answers Key

EXERCISES

Single Correct Answer Type

1. (1)	2. (2)	3. (3)	4. (3)	5. (2)
6. (3)	7. (4)	8. (2)	9. (4)	10. (3)
11. (1)	12. (3)	13. (1)	14. (2)	15. (2)
16. (3)	17. (4)	18. (2)	19. (2)	20. (3)
21. (1)	22. (2)	23. (3)	24. (3)	25. (4)
26. (3)	27. (1)	28. (1)	29. (3)	30. (4)
31. (4)	32. (3)	33. (3)	34. (4)	35. (2)
36. (2)	37. (1)	38. (3)	39. (2)	40. (3)
41. (4)	42. (3)	43. (1)	44. (4)	45. (3)
46. (1)	47. (4)	48. (2)	49. (4)	50. (2)
51. (3)	52. (3)	53. (3)	54. (2)	55. (2)
56. (1)	57. (3)	58. (2)	59. (1)	60. (4)
61. (1)	62. (3)	63. (2)	64. (4)	65. (1)
66. (4)	67. (1)	68. (3)	69. (2)	70. (2)
71. (4)	72. (1)	73. (2)	74. (4)	75. (2)
76. (1)	77. (1)	78. (2)	79. (4)	80. (1)
81. (2)	82. (4)	83. (1)		

Multiple Correct Answers Type

1. (1),(2),(3)	2. (2),(4)	3. (1),(3)
4. (1),(2)	5. (1),(3),(4)	6. (1),(2),(3),(4)
7. (3),(4)	8. (2),(4)	9. (1),(2),(3)
10. (1),(3)	11. (1),(3),(4)	12. (1),(3),(4)
13. (1),(3)		

Linked Comprehension Type

1. (2)	2. (1)	3. (1)	4. (4)	5. (2)
6. (3)	7. (3)	8. (1)	9. (2)	10. (1)
11. (1)	12. (2)	13. (1)	14. (4)	15. (3)
16. (4)	17. (2)	18. (1)	19. (2)	20. (1)

21. (3)	22. (3)	23. (4)	24. (3)	25. (2)
26. (1)	27. (2)	28. (3)	29. (2)	30. (3)
31. (1)	32. (3)	33. (2)	34. (3)	35. (2)
36. (2)	37. (1)	38. (4)	39. (1)	

Matrix Match Type

1. i. $\rightarrow$ a., c.; ii. $\rightarrow$ b., d.; iii. $\rightarrow$ d.; iv. $\rightarrow$ a., c.
2. i. $\rightarrow$ d.; ii. $\rightarrow$ a., b., c., d.; iii. $\rightarrow$ b., c., d.; iv. $\rightarrow$ a., c., d.
3. i. $\rightarrow$ c.; ii. $\rightarrow$ a., b.; iii. $\rightarrow$ a., b.; iv. $\rightarrow$ d.
4. i. $\rightarrow$ a., c.; ii. $\rightarrow$ a., c.; iii. $\rightarrow$ b.; iv. $\rightarrow$ d.
5. i. $\rightarrow$ a., b., c., d.; ii. $\rightarrow$ a., b., c., d.; iii. $\rightarrow$ a., c.; iv. $\rightarrow$ a.

Numerical Value Type

1. (1)	2. (4)	3. (5)	4. (4)	5. (1)
6. (1)	7. (3)	8. (9)	9. (2)	10. (38)
11. (2.00)	12. (0.50)	13. (0.148)	14. (0.63)	15. (450)
16. (23.55)				

ARCHIVES

JEE Advanced

Single Correct Answer Type

1. (2)	2. (1)	3. (2)	4. (2)	5. (1)
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Linked Comprehension Type

1. (1)	2. (2)	3. (4)
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Numerical Value Type

1. (3)	2. (7)	3. (7)	4. (1)	5. (24)
6. (1.00)	7. (6)			

Chapter 5

Concept Application Exercises

Exercise 5.1

1. For completely absorbing surface,

$$P_{\text{rod}} = \frac{I}{c} = \frac{1.4 \times 10^3}{3.0 \times 10^8} = 4.66 \times 10^{-6} \text{ Nm}^{-2}$$

2. Here, $\lambda = 632.8 \text{ nm} = 632.8 \times 10^{-9} \text{ m}$;

Power, $P = 9.42 \text{ mW} = 9.42 \times 10^{-3} \text{ W}$

- (a) Now, energy of each photon in the light beam,

$$E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{632.8 \times 10^{-9}} \\ = 3.14 \times 10^{-19} \text{ J} = \frac{3.14 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.963 \text{ eV}$$

Also, momentum of each photon in the light beam,

$$p = \frac{h}{\lambda} = \frac{6.62 \times 10^{-34}}{632.8 \times 10^{-9}} = 1.046 \times 10^{-27} \text{ kg m s}^{-1}$$

- (b) The number of photons falling on the target per second,

$$N = \frac{P}{E} = \frac{9.42 \times 10^{-3}}{3.14 \times 10^{-19}} = 3 \times 10^{16} \text{ s}^{-1}$$

- (c) The required speed of the hydrogen atom,

$$v = \frac{p}{m_H} = \frac{1.046 \times 10^{-27}}{1.66 \times 10^{-27}} = 0.63 \text{ m s}^{-1}$$

3. Here, $h = 6.63 \times 10^{-34} \text{ Js}$; $\lambda = 1.0 \text{ nm} = 10^{-9} \text{ m}$

- (a) The electron and the proton will possess the same momentum, which is given by

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{10^{-9}} = 6.63 \times 10^{-25} \text{ kg m s}^{-1}$$

- (b) The energy of photon,

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{10^{-9}} = 1.99 \times 10^{-16} \text{ J} \\ = \frac{1.99 \times 10^{-16}}{1.6 \times 10^{-19}} = 1243.1 \text{ eV}$$

- (c) The kinetic energy of electron,

$$E = \frac{p^2}{2m} = \frac{(6.63 \times 10^{-25})^2}{2 \times 9.1 \times 10^{-31}} = 2.415 \times 10^{-19} \text{ J} \\ = \frac{2.415 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.51 \text{ eV}$$

4. Energy incident on the plate = $\frac{1200 \times 100}{60} = 2000 \text{ J s}^{-1}$

$$\therefore nh\nu = 2000$$

...(i)

$$\text{Initial momentum of photons} = \frac{nh\nu}{c}$$

$$\text{Final momentum of photons} = \frac{40}{100} \times \frac{nh\nu}{c} = \frac{0.4nh\nu}{c}$$

Force acting on the plate = Rate of change of momentum

$$= \frac{0.4nh\nu}{c} - \left(-\frac{nh\nu}{c} \right) = \frac{1.4nh\nu}{3 \times 10^8} \quad \text{...(ii)}$$

Substituting the value of $nh\nu$ from Eq. (i) in Eq. (ii), we get the force acting on the plate = $1.86 \times 10^{-5} \text{ N}$

5. Energy of photon,

$$E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{600 \times 10^{-9}} \approx 3.3 \times 10^{-19} \text{ J}$$

The number of photons emitted per second,

$$n = \frac{200}{3.3 \times 10^{-19}} = 6.1 \times 10^{20} \text{ s}^{-1}$$

The radiation is spherically symmetrical, the number of photons entering the sensor per second is the product of n and ratio of aperture area to the area of sphere of radius 10 m

$$(6.1 \times 10^{20}) \frac{\lambda \times (0.01)^2}{4 \times \pi \times 10^2} = 1.53 \times 10^{14} \text{ photon s}^{-1}$$

Therefore, number of photons entering the sensor in 1 s

$$= (0.1) \times (1.53 \times 10^{14}) = 1.53 \times 10^{13}$$

6. We know that force is rate of change of momentum: $F = \frac{\Delta p}{\Delta t}$

Since the photons are incident normally on a perfectly reflecting screen, Δp is twice of incident momentum. If n is the number of photons per second.

$$F = \frac{\Delta p}{\Delta t} = n \times 2 \times \frac{h}{\lambda} = 1 \text{ N} \quad \left(\text{since } p = \frac{h}{\lambda} \right)$$

$$\therefore n = \frac{\lambda}{2h} = \frac{662 \times 10^{-9}}{2 \times (6.62 \times 10^{-34})} = 5 \times 10^{26} \text{ photons}$$

7. (a) As energy flux, $I = [E/(S \times t)] = [P/S]$

$$P = I \times S = (1.4 \times 10^3) \times (8 \times 20) = 2.24 \times 10^5 \text{ W} = 224 \text{ kW}$$

- (b) As power $P = (E/t)$,

$$E = P \times t = (2.24 \times 10^5) \times (60 \times 60) \\ = 8.064 \times 10^8 = 806.4 \text{ MJ}$$

- (c) For perfectly absorbing surface radiation pressure,

$$P = \frac{I}{c} = \frac{1.4 \times 10^3}{3 \times 10^8} = 4.7 \times 10^{-6} \text{ Nm}^{-2} \text{ and as } P = (F/S)$$

$$F = P \times S = 4.7 \times 10^{-6} \times (8 \times 20) = 7.52 \times 10^{-4} \text{ N}$$

8. For equilibrium, the force exerted by the light beam should balance the weight of plate.

$$F_{\text{photon}} = mg \quad \left(F_{\text{photon}} = \frac{IA}{c} = \frac{P}{c}, \text{ where power } P = IA \right)$$

$$\Rightarrow \frac{P}{c} = 10 \times 10^{-3} \times 10 \Rightarrow P = 3 \times 10^7 \text{ W}$$

9. $I = 1400 \text{ Wm}^{-2}$; $\lambda = 6000 \text{ \AA}$

- (a) Energy of the photon, $E = hf = \frac{hc}{\lambda}$ ($c = 3 \times 10^8 \text{ m s}^{-1}$)

Let n be the number of photons received per second per unit area.

$$\therefore n = \frac{IA}{E} = \frac{(1400 \times 1) \times (6000 \times 10^{-10})}{6.63 \times 10^{-34} \times 3 \times 10^8} \\ = 4.22 \times 10^{21}$$

(b) Total energy emitted per second = power (W)

Number of photons emitted from the sun per second,

$$n = \frac{\text{Power of sun (W)}}{\text{Energy of a photon}} = \frac{I \times (4\pi(1.49 \times 10^{11})^2) \times (6000 \times 10^{-10})}{6.63 \times 10^{-34} \times 3 \times 10^8} = 1.178 \times 10^{45}$$

(R : average radius of earth's orbit)

10. Energy of one photon, $E = hc/\lambda$

If power of source is P , the number of photons incident on the metallic surface is

$$N = \frac{P}{E} = \frac{\lambda P}{hc}$$

Momentum of incident photons = h/λ

Change in momentum due to reflection = $2h/\lambda$

The total momentum imparted to the surface per second is

$$\frac{2h}{\lambda} \frac{\lambda P}{hc} = \frac{2P}{c}$$

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \left(\frac{2P}{c} \right) / A = \frac{2P}{cA} = \frac{2 \times 5 \times 10^{-3}}{3 \times 10^8 \times 10^{-6}} = 3.33 \times 10^{-5} \text{ N m}^{-2}$$

11. The momentum of a photon, $p = \frac{hf}{c} = \frac{E}{c}$

Momentum of reflected photon = $\rho \left(\frac{E}{c} \right)$

$$\therefore \text{Change of momentum} = \left(\frac{E}{c} \right) + \left(\frac{\rho E}{c} \right) = \frac{(1+\rho)E}{c}$$

$$\text{Force exerted, } F = \frac{\left[(1+\rho) \frac{E}{c} \right]}{T}$$

$$\text{Pressure, } P = \frac{F}{A} = \frac{4 \left[(1+\rho) \frac{E}{c} \right]}{T\pi d^2} = \frac{4(1+\rho)E}{\pi c T d^2} = \frac{4(1+0.5) \times 10}{\pi \times 3 \times 10^8 \times 10^{-4} \times 10^{-10}} = 6.37 \times 10^6 \text{ N m}^{-2}$$

12. If S is the area of the mirror on which light falls, the transverse section of the incident beam is $S \cos \theta$. Momentum of the incident

photons per second, $p = \left(\frac{I}{c} \right) S \cos \theta$

Normal component of momentum incident per second

$$= (p_n)_{\text{incident}} = \left(\frac{IS \cos \theta}{c} \right) \cos \theta = \frac{IS \cos^2 \theta}{c}$$

Momentum of reflected photons per second

$$(p_n)_{\text{reflected}} = \rho \left(\frac{IS \cos^2 \theta}{c} \right) \text{ in the opposite direction.}$$

$$\therefore \text{Rate of change of momentum} = \text{force} = \frac{\Delta p}{\Delta t} = \frac{(1+\rho)IS \cos^2 \theta}{c}$$

$$\therefore \text{Normal pressure } P = \frac{F}{A} = \frac{(1+\rho)I \cos^2 \theta}{c}$$

$\Rightarrow$ Required pressure,

$$P = \frac{(1+0.5)(0.5 \times 10^4 \cos^2 45^\circ)}{3 \times 10^8} = 1.25 \times 10^{-5} \text{ N m}^{-2}$$

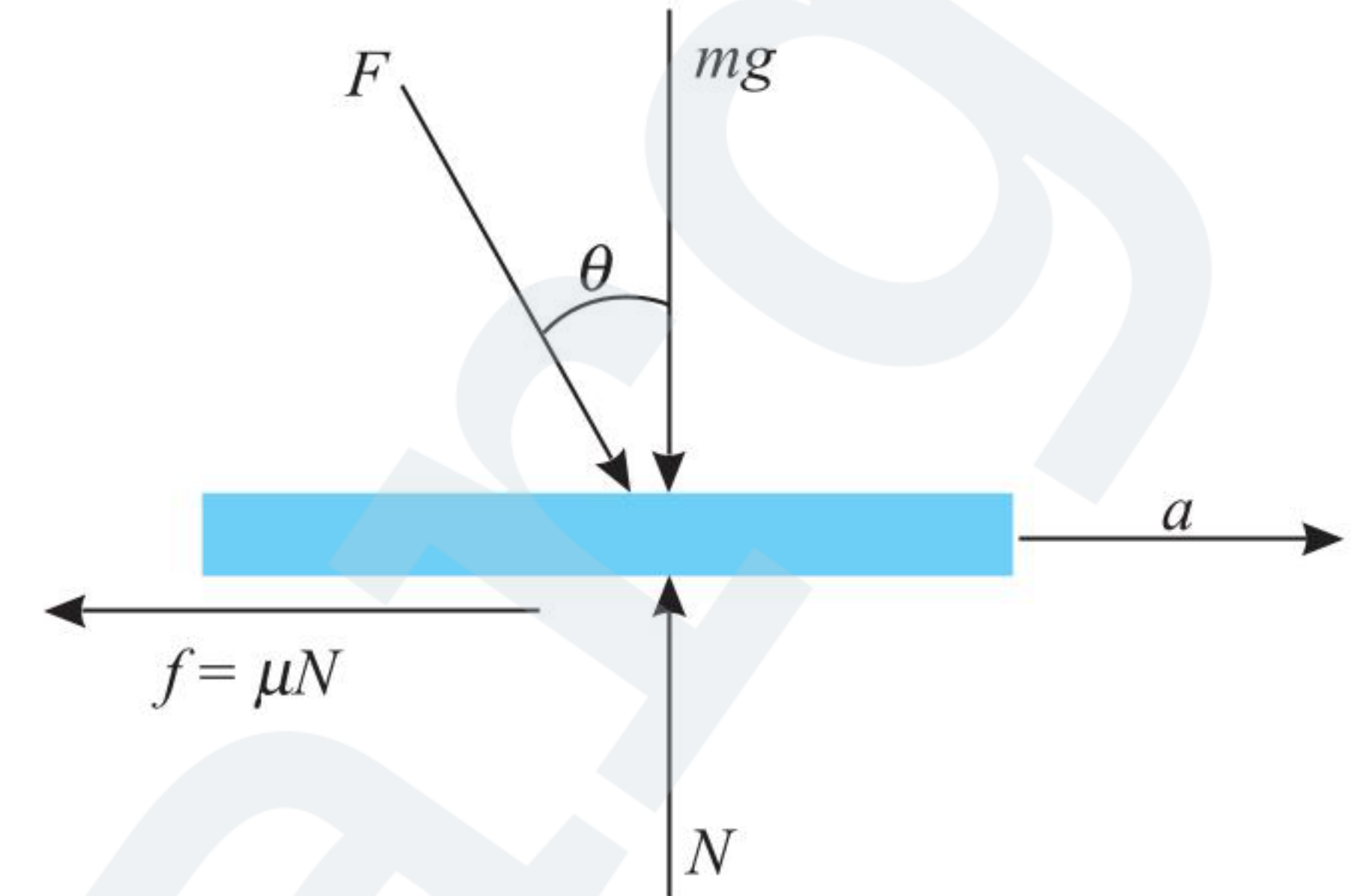
13. E = Energy received by surface per second is $IA \cos \theta = Iab \cos \theta$

Number of photons received by the surface per second,

$$n = \frac{E}{hc/\lambda} = \frac{Iab \cos \theta \lambda}{hc}$$

$$\Delta p_{\text{one photon}} = \frac{h}{\lambda} \Rightarrow F_{\text{radiation}} = \Delta p_{\text{one photon}} \cdot \lambda = \frac{Iab \cos \theta}{c}$$

Let us draw the FBD of the plank (see figure)



$$F \sin \theta - \mu N = m\alpha \quad \dots(i)$$

$$N = mg + F \cos \theta \quad \dots(ii)$$

$$\text{From Eqs. (i) and (ii), we get } \alpha = \frac{F \sin \theta - \mu(mg + F \cos \theta)}{m}$$

Exercise 5.2

1. The energy of incident photon, $E = \frac{12400}{4800} = 2.58 \text{ eV}$

$$\Rightarrow KE_{\text{max}} = E - \phi = 2.58 - 2.30 = 0.28 \text{ eV}$$

$$\text{Threshold wavelength } \lambda = \frac{12400}{2.30} = 5391.3 \text{ \AA}$$

2. Energy of incident photon, $E = \frac{12400}{1500} \text{ eV} = 8.26 \text{ eV}$

According to photo electric effect equation,

$$E = \phi + KE_{\text{max}} \Rightarrow \phi = E - KE_{\text{max}}$$

$$\Rightarrow \phi = 8.26 \text{ eV} - 3 \text{ eV} = 5.26 \text{ eV}$$

Threshold wavelength for the metal surface corresponding to work function 5.29 eV is given as

$$\lambda_{\text{th}} = \frac{12400}{5.26} \text{ \AA} = 2357.4 \text{ \AA}$$

Stopping potential for the ejected electrons can be given as

$$V_0 = \frac{KE_{\text{max}}}{e} = 3 \text{ V}$$

3. We use

$$eV_s = \frac{hc}{\lambda} - \phi = \frac{1240(\text{nm})\text{eV}}{400(\text{nm})} - 1.9 \text{ eV}$$

$$\Rightarrow eV_s = 1.2 \text{ eV}$$

$$\Rightarrow V_s = 1.2 \text{ V}$$

Thus the cesium ball can be charged to a maximum potential of 1.2 V.

4. The energy of each photon of incident light is

$$E = h\nu = \frac{hc}{\lambda} = \frac{(6.62 \times 10^{-34})(3 \times 10^8)}{(4560 \times 10^{-10})} = 4.35 \times 10^{-19} \text{ J}$$

Number of photons in one milliwatt source

$$N = \frac{10^{-3}}{4.35 \times 10^{-19}} = 2.29 \times 10^{15} \text{ s}$$

(As power of source = 1 milliwatt = 10^{-3} W)

$$\text{Number of photons released, } N_{\text{th}} = (2.29 \times 10^{15}) \times \frac{0.5}{100} \\ = 1.14 \times 10^{13} / \text{s}$$

$$\text{Thus photo-electric current, } I = (1.14 \times 10^{13})(1.6 \times 10^{-9}) \\ = 1.824 \times 10^{-6} \text{ A} = 1.824 \mu\text{A}$$

$$5. W_0 = 4.5 \text{ eV, } \lambda = 200 \text{ nm}$$

$$\text{Stopping potential or energy, } E - W_0 = \frac{hc}{\lambda} - W$$

Minimum 1.7 V is necessary to stop the electron

The minimum KE = 2 eV

[Since the electric potential of 2 V is required to accelerate the electron to reach the plates] the maximum KE = (2 + 1.7) eV = 3.7 eV.

6. Let W_0 be the work function of the metal. Then maximum kinetic energies of the photoelectrons ejected in the two cases will be,

$$eV_1 = hf_1 - W_0$$

$$\text{And, } eV_2 = hf_2 - W_0 \therefore e(V_2 - V_1) = h(f_2 - f_1)$$

$$\text{or, } h = \frac{e(V_2 - V_1)}{f_2 - f_1}$$

If f_0 is the threshold frequency, then $eV_1 = hf_1 - W_0 = hf_1 - hf_0$

$$\text{or } f_0 = f_1 - \frac{eV_1}{h} = f_1 - \frac{V_1(f_2 - f_1)}{V_2 - V_1} \\ = \frac{f_1V_2 - f_1V_1 - f_2V_1 + f_1V_1}{V_2 - V_1} = \frac{f_1V_2 - f_2V_1}{V_2 - V_1}$$

7. Let W_0 be the work function of the metal. Let E_1 and E_2 be the kinetic energies of photoelectrons corresponding to frequencies f and $2f$ of the incident radiation.

Using Einstein's photoelectric equation, we find $hf = E_1 + W_0$

$$\text{and } 2hf = E_2 + W_0$$

$$\text{On dividing, } 2 = \frac{E_2 + W_0}{E_1 + W_0}$$

$$\text{or } 2E_1 + 2W_0 = E_2 + W_0$$

$$\text{or } E_2 = 2E_1 + W_0$$

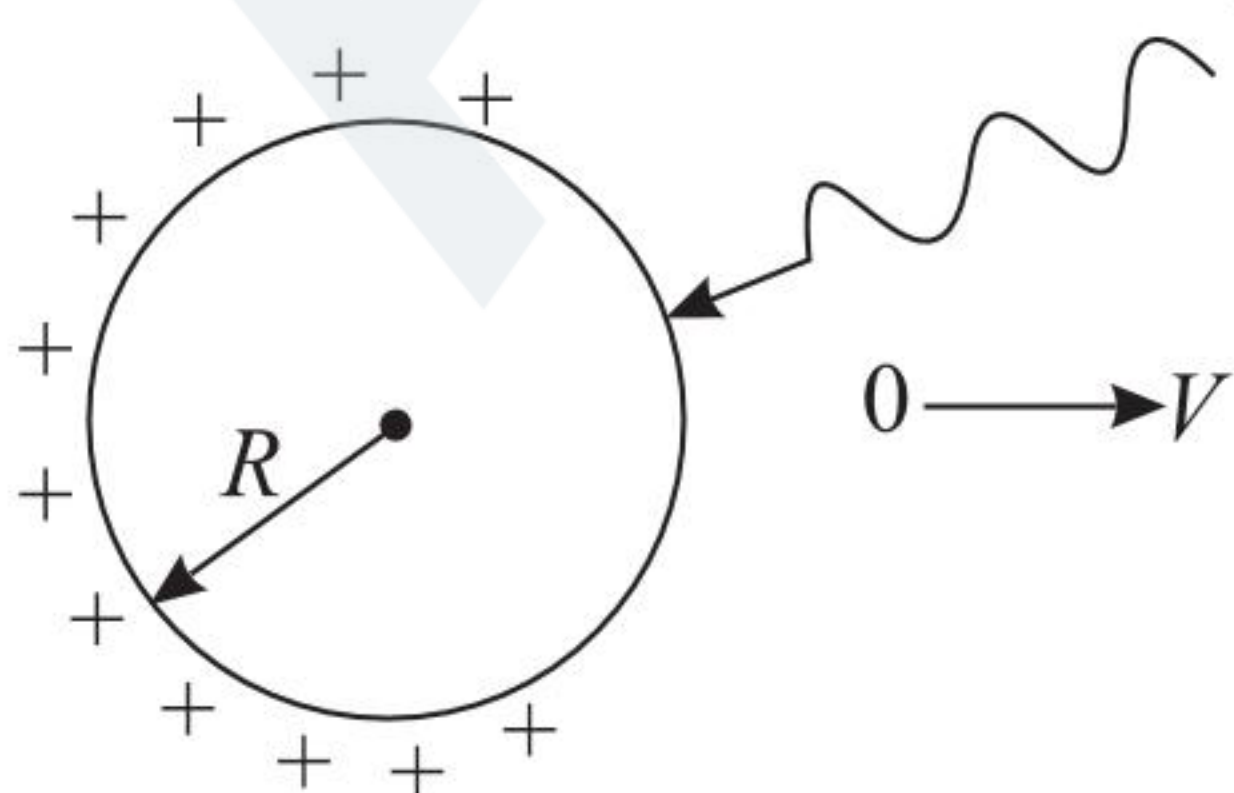
Thus kinetic energy of photoelectrons is increased more than double on doubling the frequency of incident radiation.

8. As the copper sphere loses photoelectrons, it becomes positively charged. Let it lose n electrons. Then its charge will be ne . Hence, the potential of the copper ball is

$$V_{\text{stop}} = \frac{1}{4\pi\epsilon_0} \frac{ne}{R} \quad \dots(i)$$

This is the stopping potential which is equal to the maximum KE of the photoelectrons.

According to Einstein's photoelectric Eq. (i), $\frac{hc}{\lambda} - W_0 = \frac{1}{2}mv_{\text{max}}^2$



$$\text{or } \frac{hc}{\lambda} - W_0 = eV_{\text{stop}} \quad \dots(ii)$$

$$\text{By using Eqs. (i) and (ii), } \frac{hc}{\lambda} - W_0 = e \left(\frac{1}{4\pi\epsilon_0} \right) \frac{ne}{R}$$

$$\text{or } n = \left(\frac{hc}{\lambda} - W_0 \right) \frac{4\pi\epsilon_0 R}{e^2}$$

On substituting corresponding values we get $n = 4.86 \times 10^9$

9. The electron ejected with maximum speed v_{max} are stopped by electric field $E = 4 \text{ N/C}$ after travelling a distance $d = 1 \text{ m}$. Thus we have

$$\frac{1}{2}mv_{\text{max}}^2 = eEd = 4 \text{ eV}$$

$$\text{The energy of incident photon} = \frac{12400}{2000} = 6.2 \text{ eV}$$

From equation of photo electric effect we use

$$\frac{1}{2}mv_{\text{max}}^2 = h\nu - \phi_0 \Rightarrow \phi_0 = 6.2 - 4 = 2.2 \text{ eV}$$

Exercise 5.3

$$1. (i) \text{ de-Broglie wavelength, } \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$$

$$\text{For same } V, \lambda \propto \frac{1}{\sqrt{mq}} \therefore \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}}$$

$$\therefore \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = \sqrt{\frac{4m_p \cdot 2e}{m_p \cdot e}} = \sqrt{8} = 2\sqrt{2}$$

Hence $\lambda_p > \lambda_\alpha$ i.e., proton has a greater value of de-Broglie wavelength.

(ii) Kinetic energy, $K = qV$

For same V , $K \propto q$

$$\therefore \frac{K_p}{K_\alpha} = \frac{q_p}{q_\alpha} = \frac{e}{2e} = \frac{1}{2}$$

Hence, $K_p < K_\alpha$, i.e., proton has less kinetic energy.

$$2. (i) \text{ de-Broglie wavelength, } \lambda = \frac{h}{\sqrt{2mqV}}$$

$$\text{For same } V, \lambda \propto \frac{1}{\sqrt{mq}}$$

$$\therefore \frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p \times e}{m_e \times e}} = \sqrt{\frac{m_p}{m_e}}$$

As $m_p > m_e$, so $\lambda_e > \lambda_p$ i.e., electron has a greater de-Broglie wavelength.

(ii) Momentum, $p = \sqrt{2meV}$

$$\text{or } p \propto \sqrt{m}$$

As $m_e < m_p$, $p_e < p_p$ i.e., electron has less momentum.

$$3. \text{ The kinetic energy of a particle, } K = \frac{1}{2}mv^2 = \frac{1}{2} \frac{(mv)^2}{m} = \frac{p^2}{2m}$$

$$\therefore \text{ Linear momentum, } p = \sqrt{2mK}$$

$$\text{de-Broglie wavelength, } \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$$

$$\text{For the particles possessing same kinetic energy, } \lambda \propto \frac{1}{\sqrt{m}}$$

$$\text{As } m_e \ll m_p < m_\alpha \therefore \lambda_e \gg \lambda_p > \lambda_\alpha$$

Hence the α -particle has the shortest de-Broglie wavelength.

4. The kinetic energy gained by a charged particle when accelerated through a potential difference of V volts is given by

$$\frac{1}{2}mv^2 = qV$$

$$\text{or } m^2 v^2 = 2mqV$$

$$\text{or } p = mv = \sqrt{2mqV}$$

$$\therefore \text{ de Broglie wavelength, } \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$$

For the same potential difference V ,

$$\frac{\lambda(\alpha\text{-particle})}{\lambda(\text{proton})} = \sqrt{\frac{m_p q_p}{m_\alpha q_\alpha}} = \sqrt{\frac{m_p \times e}{4m_p \times 2e}} = \frac{1}{2\sqrt{2}}$$

5. If α particle is accelerated through a potential difference V , then

$$qV = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$\text{or } p = \sqrt{2mqV}$$

$$\therefore \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$$

$$\text{As } \lambda_p = \lambda_\alpha$$

$$\text{or } \frac{h}{\sqrt{2m_p q_p V_p}} = \frac{h}{\sqrt{2m_\alpha q_\alpha V_\alpha}}$$

$$\text{or } \frac{V_p}{V_\alpha} = \frac{m_\alpha q_\alpha}{m_p q_p} = \frac{4m_p \cdot 2e}{m_p \cdot e} = 8$$

$$6. \lambda_{\text{de Broglie}} = \frac{h}{\sqrt{2mK}} = \frac{h}{mv}$$

$$v = u + at = 0 + \left(\frac{eE}{m}\right)t \Rightarrow \lambda = \frac{h}{eEt}$$

$$\Rightarrow \frac{d\lambda}{dt} = -\frac{h}{eEt^2}$$

$$7. \lambda_{\text{de Broglie}} = \frac{h}{mV_{\text{mp}}} \quad \dots(i)$$

$$V_{\text{mp}} = \sqrt{\frac{2kT}{m}} \quad \dots(ii)$$

$$\begin{aligned} \text{Then, } \lambda_{\text{de Broglie}} &= \frac{h}{m\sqrt{\frac{2kT}{M}}} \\ &= \frac{6.63 \times 10^{-34}}{\left(\sqrt{\frac{2 \times 1.38 \times 10^{-23} \times 300}{2 \times 1.67 \times 10^{-27}}}\right) \times 2 \times 1.67 \times 10^{-27}} \\ &= 1.2 \times 10^{-7} \text{ m} \end{aligned}$$

Exercises

Single Correct Answer Type

$$1. (1) \text{ Energy} = \frac{1}{2}mv^2 = 5000 \text{ eV} = 5000 \times 1.6 \times 10^{-19} \text{ J}$$

$$mv = \sqrt{2 \times 5000 \times (1.6 \times 10^{-19})m} = 4 \times 10^{-8} \times \sqrt{m}$$

Number of electrons striking per second is

$$n = \frac{q}{e} = \frac{It}{e} = \frac{50 \times 10^{-6} \times 1}{1.6 \times 10^{-19}} = 31.25 \times 10^{13}$$

Force = change of momentum per second

$$= n(mv) = 31.25 \times 10^{13} \times 4 \times 10^{-8} \sqrt{m}$$

$$= 125 \times 10^5 \sqrt{9.1 \times 10^{-31}}$$

$$= 1.1924 \times 10^{-8} \text{ N}$$

2. (2) Number of photons falling per second.

$$N_p = \frac{10^{-3}}{\frac{6.6 \times 10^{-34} \times 3 \times 10^8}{5000 \times 10^{-10}}} = 2.5 \times 10^{15}$$

Let N_e is the number of photoelectrons emitted per second.

$$\therefore I = \frac{q}{t} = \frac{N_e e}{1} \Rightarrow N_e = \frac{I}{e} = \frac{0.16 \times 10^{-6}}{1.6 \times 10^{-19}} = 10^{12}$$

Percentage of photons producing photoelectrons,

$$= \frac{N_e}{N_p} \times 100 = \frac{10^{12}}{2.5 \times 10^{15}} \times 100 = 0.04\%$$

3. (3) Using photoelectric equation, $hf - hf_0 = \frac{1}{2}mv^2 = eV_s$

$$\text{or } \left(\frac{hc}{\lambda} - \frac{hc}{\lambda_0}\right) = eV_s$$

$$\text{For the first case, } \frac{hc}{\lambda} - \frac{hc}{\lambda_0} = e(3V_0) \quad \dots(i)$$

$$\text{For the second case, } \frac{hc}{2\lambda} - \frac{hc}{\lambda_0} = e(V_0) \quad \dots(ii)$$

Solving $\lambda_0 = 4\lambda$

4. (3) Einstein's equation for photoelectric effect is

$$hf - hf_0 = \frac{1}{2}mv_{\text{max}}^2$$

$$\text{When } f = 2f_0, f_{\text{max}} = 4 \times 10^8 \text{ cm s}^{-1}$$

$$2hf_0 - hf_0 = (1/2)m(4 \times 10^8)^2$$

$$hf_0 = \frac{1}{2}m(4 \times 10^8)^2 \quad \dots(i)$$

$$\text{When } f = 5f_0, v_{\text{max}} = v'$$

$$h(5f_0) - hf_0 = \frac{1}{2}mv'^2 \quad \dots(ii)$$

Dividing Eq. (ii) by Eq. (i), we get $v' = 8 \times 10^8 \text{ cm s}^{-1}$

$$5. (2) E - W_0 = \frac{1}{2}mv^2 = eV_s$$

$$\text{or } \frac{hc}{\lambda} - W_0 = eV_s$$

$$\text{Hence, } \frac{hc}{0.6 \times 10^{-6}} - W_0 = e(0.5) \quad \dots(i)$$

$$\text{and } \frac{hc}{0.4 \times 10^{-6}} - W_0 = e(1.5) \quad \dots(ii)$$

Solving, we get $W_0 = 1.5 \text{ eV}$

$$6. (3) n = \frac{\text{power}}{hc/\lambda} = \frac{1 \times 300 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^8}$$

$$= 1.51 \times 10^{18} \text{ m}^{-2} \text{ s}^{-1} = 1.51 \times 10^{14} \text{ cm}^{-2} \text{ s}^{-1}$$

As only 1 percent of photons cause emission of photoelectrons, number of photoelectrons is

$$n_e = 1.51 \times 10^{12} \text{ s}^{-1}$$

$$7. (4) \frac{1}{2}mv^2 = \frac{hc}{\lambda} - W_0 \quad \dots(i)$$

Let the speed of the fastest electron be v_1 when excitation wavelength is changed to $3\lambda/4$.

$$\therefore \frac{1}{2}mv_1^2 = \frac{4hc}{3\lambda} - W_0$$

$$\Rightarrow \frac{1}{2}mv_1^2 = \frac{4}{3}\left(\frac{hc}{\lambda} - W_0\right) + \frac{W_0}{3}$$

$$\Rightarrow \frac{1}{2}mv_1^2 = \frac{4}{3}\left(\frac{1}{2}mv^2\right) + \frac{W_0}{3} \quad [\text{using Eq. (i)}]$$

$$\Rightarrow v_1^2 = \frac{4v^2}{3} + \frac{2W_0}{3m}$$

$$\therefore v_1 > \sqrt{\frac{4}{3}} v$$

$$8. (2) hf_1 - hf_0 = \frac{1}{2}mv_1^2$$

$$hf_2 - hf_0 = \frac{1}{2}mv_2^2$$

$$\therefore h(f_1 - f_2) = \frac{1}{2}m(v_1^2 - v_2^2)$$

$$\therefore v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$$

9. (4) Since the number of photoelectrons emitted is directly proportional to the intensity of incident radiation, the number of photoelectrons emitted becomes four times.

The energy of photoelectrons does not change with the intensity of light.

$$10. (3) \frac{hc}{\lambda_{\max}} = 3 \times 1.6 \times 10^{-19} \text{ J}$$

$$\Rightarrow \lambda_{\max} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{3 \times 1.6 \times 10^{-19}} = 4.125 \times 10^{-7} \text{ m}$$

$$11. (1) K_{\max} = h\nu - W$$

ω is the intercept on y -axis and h is the slope.

$$\therefore h = \frac{2.4 \times 10^{-15}}{4 \times 10^{18}} = 6 \times 10^{-34} \text{ Js}$$

$$W = 2 \times 10^{-15} \text{ J}$$

$$\Rightarrow h\nu_0 = 2 \times 10^{-15}$$

$$\text{or } \nu_0 = 3.33 \times 10^{18} \text{ Hz}$$

12. (3) According to Einstein's equation, $E = W_0 + K$

$$W_{0\max} = 4.2 \text{ eV}$$

$$K = 2.6 \text{ eV}$$

$$\therefore E_{\min} = W_{0\max} + K = (4.2 + 2.6) \text{ eV} = 6.8 \text{ eV}$$

13. (1) As work function $W = h\nu_0$, where ν_0 is the threshold frequency. Greater the work function, greater is the threshold frequency. Therefore, the threshold frequency of sodium will be lesser than that for aluminium.

14. (2) The maximum KE of the photoelectron is given by

$$\left(\frac{1}{2}mv^2\right)_{\max} = hf - W$$

$$\text{Now, } f = \frac{c}{\lambda} \quad \text{and} \quad \left(\frac{1}{2}mv^2\right) = eV$$

$$\therefore eV = \frac{hc}{\lambda} - W$$

$$\text{or } V = \left(\frac{hc}{e}\right) \frac{1}{\lambda} - \frac{W}{e} \quad \dots(i)$$

Slope of straight line = hc/e

15. (2) Since V is represented along y -axis and $(1/\lambda)$ along x -axis, Eq. (i) represents a straight line.

Intercept of straight line = $-(W/e)$

16. (3) The ratio of electric force to magnetic force is

$$\frac{F_e}{F_B} = \frac{qE}{qvB} = \frac{3 \times 10^6}{2.4 \times 10^6 \times 1.5} = \frac{5}{6}$$

Now, magnetic force on beta particles acts downward, whereas electric force acts upward and both are in the plane of diagram. But since magnetic force is larger, so beta particles are deflected downward.

17. (4) By photoelectric equation: $\frac{1}{2}mv^2 = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$, where λ is the wavelength of incident radiation and λ_0 is the threshold wavelength.

$$\frac{1}{2}m(2v)^2 = hc\left(\frac{1}{\lambda'} - \frac{1}{\lambda_0}\right)$$

Solving for λ' , we get $\lambda' = 300 \text{ nm}$

18. (2) By Einstein's particle (photon) theory, the maximum kinetic energy $E_{\max}$ of the emitted electrons from the cathode is proportional to the frequency f of the light. This is expressed in the Einstein's photoelectric equation below

$$E_{\max} = hf - \phi = hf - hf_0 = h(f - f_0) \quad \dots(i)$$

where h is the Planck's constant, ϕ is the work function of the metal and is related to the threshold frequency f_0 by $\phi = hf_0$. It is the minimum amount of work or energy necessary to take a free electron out of the metal against the attractive forces of surrounding positive ions.

At a particular negative potential difference V applied to the anode A , the current becomes zero. This is value of the negative potential difference which just stops the electrons with maximum energy from reaching A . V is called the stopping potential. Therefore,

$$eV = E_{\max} \quad \dots(ii)$$

From Eqs. (i) and (ii), we have $eV = E_{\max} = h(f - f_0)$

$$V = \frac{h}{e}(f - f_0) \text{ or } y = \frac{h}{e}(x - x_0)$$

The variation of V (or y) is thus a straight line of gradient h/e , when it is plotted against f (or x) at f_0 . It is best represented in graph (2).

19. (2) On increasing intensity, only saturation current increases, whereas retarding potential remains the same because wavelength of light is unchanged.

$$20. (3) eV_s = \frac{hc}{\lambda} - \phi_0 \text{ or } eV_s + \phi_0 = \frac{hc}{\lambda}$$

$$\text{or } \lambda = \frac{hc}{eV_s + \phi_0}$$

$$\Rightarrow \frac{\lambda_2}{\lambda_1} = \frac{eV_{s1} + \phi_0}{eV_{s2} + \phi_0}$$

$$\text{or } \frac{4500}{4000} = \frac{1.3 + \phi_0}{0.9 + \phi_0}$$

$$\text{or } \phi_0 = 851.3 - 9 \times 0.9$$

Solving, $\phi_0 = (10.4 - 8.1) = 2.3 \text{ eV}$

21. (1) The photoelectrons will be emitted because wavelength of incident radiation is less than threshold wavelength ($\lambda < \lambda_0$)

$$22. (2) \left(\frac{1}{2}mv^2\right)_{\max} = hf - W$$

When f is doubled (W remains same), $\left(\frac{1}{2}mv^2\right)_{\max}$ i.e., (KE) is

increased. The photoelectric current is directly proportional to the intensity of incident light.

23. (3) Let $hf_0 - W_0 = K$... (i)

If frequency is doubled, let kinetic energy of photoelectrons be K_1 .

$2hf_0 - W_0 = K_1$... (ii)

$\Rightarrow 2(hf - W_0) + W_0 = K_1$

$\Rightarrow 2K + W_0 = K_1$

i.e., kinetic energy is more than doubled.

24. (3) If E is the energy of incident photon and W_0 the work function, then $E - W_0 =$ available energy.

$E - W_0 = \frac{1}{2}mv^2$

or $v = \sqrt{\frac{2(E - W_0)}{m}}$

$\therefore \frac{v_1}{v_2} = \sqrt{\frac{1 - 0.5}{2.5 - 0.5}} = \sqrt{\frac{0.5}{2}} = \frac{1}{2}$

25. (4) $\lambda_p = \lambda_\alpha$

or $\frac{h}{\sqrt{2m_p Q_p V}} = \frac{h}{\sqrt{2m_\alpha Q_\alpha V_\alpha}}$

$\therefore m_p Q_p V_p = m_\alpha Q_\alpha V_\alpha$

$\therefore V_\alpha = \left(\frac{m_p}{m_\alpha}\right)\left(\frac{Q_p}{Q_\alpha}\right)V = \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)V = \frac{V}{8}$

26. (3) $I \propto 1/d^2$

When source is placed 2 m away, then $I' = (I/4)$. The number of electrons emitted $\propto$ intensity. Hence, the number of emitted electrons is reduced to one-fourth.

27. (1) Energy radiated as visible light is $\frac{5}{100} \times 100 = 5 \text{ J s}^{-1}$

Let n be the number of photons emitted per second.

Then, $\frac{nhc}{\lambda} = 5$

$\therefore n = \frac{5\lambda}{hc} = \frac{5 \times 5.6 \times 10^{-7}}{(6.62 \times 10^{-34})(3 \times 10^8)} = 1.4 \times 10^{19}$

28. (1) $hf = 5 \text{ eV} + 2.2 \text{ eV} = 7.2 \text{ eV}$

$7.2 = \frac{12375}{\lambda(\text{in } \text{\AA})}$

or $\lambda(\text{in } \text{\AA}) = \frac{12375}{7.2} \approx 1719$

29. (3) $\phi_0 = hf_0 = \frac{hc}{\lambda_0}$

$\phi_0 \lambda_0 = \text{constant}$

$4.5 \times \lambda = 2.3 \times 5460$

or $\lambda = \frac{2.3 \times 5460}{4.5} \text{\AA} = 2790.7 \text{\AA} \approx 2791 \text{\AA}$

30. (4) Work function, $W_0 = \frac{hc}{\lambda_0}$

$\Rightarrow W_0 = \frac{2 \times 10^{-25}}{2 \times 10^{-7}} = 10^{-18} \text{ J} = \frac{10^{-18}}{1.6 \times 10^{-19}} \text{ eV} = 6.25 \text{ eV}$

Energy of incident radiation is $E = \frac{hc}{\lambda} = 31.25 \text{ eV}$

$\therefore$ KE of photoelectrons $= E - W_0 = 25 \text{ eV}$

31. (4) $\frac{1}{2}mv_{\text{max}}^2 = \frac{hc}{\lambda} - \phi_0$

$\frac{1}{2}mv_{\text{max}}^2 = \frac{12375 \text{ eV}}{3000} - 1 \text{ eV} = 3.125 \times 1.6 \times 10^{-19}$

$v_{\text{max}} = \sqrt{\frac{2 \times 3.125 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} \approx 10^6 \text{ m s}^{-1}$

32. (3) Let us calculate energy corresponding to $0.2 \times 10^{-6} \text{ m}$

or $0.2 \times 10^{-6} \times 10^{10} \text{\AA}$

or $2000 \text{\AA} = \frac{12375}{2000} = 6.1875 \text{ eV}$

$\frac{1}{2}mv_{\text{max}}^2 = (6.1875 - 4) \text{ eV}$

or $mv_{\text{max}}^2 = \frac{2 \times 2.1875 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}$

or $mv_{\text{max}}^2 = 0.769 \times 10^{12} \Rightarrow v = 0.876 \times 10^6 = 8.76 \times 10^5 \text{ m s}^{-1}$

33. (3) Effective power $= \frac{25}{100} \times 200 \text{ W} = 50 \text{ W}$

Now, $50 = nhv = \frac{nhc}{\lambda}$

$n = \frac{50\lambda}{hc} = \frac{50 \times 0.6 \times 10^{-6}}{6.6 \times 10^{-34} \times 3 \times 10^8} = 1.5 \times 10^{20}$

34. (4) If q is the charge on the particle and V the potential difference through which it is accelerated, then

$qV = \frac{1}{2}mv^2$

or $mv = \sqrt{2mqV}$

de Broglie's wavelength, $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mqV}}$

$\therefore \frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}}$

35. (2) We know that mass m in motion and the rest mass m_0 is related through the equation

$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

As $v = c$, $m = \frac{m_0}{\sqrt{1 - 1}} = \frac{m_0}{0} = \infty$

Therefore, de Broglie wavelength is $\lambda = \frac{h}{mv} = \frac{h}{(\infty)(c)} = 0$

36. (2) $W = hf - eV_s$

$hf =$ energy of incident photon

Here $hf = \frac{12400}{1240} \text{ eV} = 10 \text{ eV}$

$\therefore W = 10 - 8 = 2 \text{ eV}$

So, $\lambda_0 =$ Threshold wavelength $= \frac{12400}{2} \text{\AA} = 6200 \text{\AA}$

37. (1) $\lambda_1 = \frac{12375}{E_1(\text{eV})} \text{\AA} = 1000 \text{\AA}$

$\therefore E_1 = 12.375 \text{ eV}$

Similarly, $E_2 = \frac{12375}{\lambda_2(\text{\AA})} \text{ eV} = \frac{12375}{2000} = 6.1875 \text{ eV}$

Now, $E_1 - W_0 = eV_s$

and $E_2 - W_0 = eV'_s$

Hence, $12.375 - W_0 = 7.7 \text{ eV}$

and $6.1875 - W_0 = eV'_s$

Solving, we get $V'_s = 1.5 \text{ V}$

38. (3) Energy is given by $E = \frac{m_0 c^2}{\sqrt{1 - (v^2/c^2)}}$

$$\text{or } E^2 = \frac{m_0^2 c^2}{c^2 - v^2}$$

$$\text{Momentum } p \text{ is given by } p = \frac{m_0 v}{\sqrt{1 - (v^2/c^2)}}$$

$$\text{or } p^2 c^2 = \frac{m_0^2 c^4 v^2}{c^2 - v^2}$$

$$\therefore E^2 - p^2 c^2 = m_0^2 c^4 \quad \text{or} \quad E^2 = p^2 c^2 + m_0^2 c^4$$

For photon, rest mass $m_0 = 0$, so $E = pc$

For electron, $m_0 \neq 0$, so $E \neq pc$

$$39. (2) \quad eV = hc \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right) \quad \dots(i)$$

$$\frac{eV}{3} = hc \left(\frac{1}{2\lambda} - \frac{1}{\lambda_0} \right) \quad \dots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get $\lambda_0 = 4\lambda$

$$40. (3) \quad \frac{hc}{\lambda} = E + \phi_0$$

$$\frac{hc}{\lambda'} = 2E + \phi_0$$

$$\text{Dividing, we get } \frac{\lambda'}{\lambda} = \left(\frac{E + \phi_0}{2E + \phi_0} \right) \quad \text{or} \quad \frac{\lambda'}{\lambda} < 1$$

$$\therefore \lambda' < \lambda \quad \text{or} \quad \lambda > \lambda' \quad \dots(i)$$

$$\text{Also, } \frac{\lambda'}{\lambda} = \frac{1}{2} \left[\frac{E + \phi_0}{E + \frac{\phi_0}{2}} \right]$$

$$\text{or } \frac{\lambda'}{\lambda} > \frac{1}{2}$$

$$\text{or } \lambda' > \frac{\lambda}{2} \quad \dots(ii)$$

It follows from Eqs. (i) and (ii) that $\lambda > \lambda' > \frac{\lambda}{2}$

$$41. (4) \quad \lambda = \frac{h}{\sqrt{2meV}}$$

$$p = \sqrt{2meV}$$

$$\frac{p_e}{p_\alpha} = \sqrt{\frac{2m_e eV}{2m_\alpha (2e)V}}$$

$$\frac{p_e}{p_\alpha} = \sqrt{\frac{m_e}{2m_\alpha}}$$

$$42. (3) \quad E = \frac{hc}{\lambda} - \phi_0$$

$$2E = \frac{hc}{\lambda'} - \phi_0$$

$$\text{Solving } \lambda' = \frac{hc\lambda}{E\lambda + hc}$$

$$43. (1) \quad E = \frac{hc}{\lambda/2} - \phi_0$$

$$2E = \frac{hc}{\lambda/3} - \phi_0$$

$$\text{or } 2 \left(\frac{2hc}{\lambda} - \phi_0 \right) = \frac{3hc}{\lambda} - \phi_0$$

$$\text{or } \frac{4hc}{\lambda} - 2\phi_0 = \frac{3hc}{\lambda} - \phi_0$$

$$\text{or } \phi_0 = \frac{hc}{\lambda}$$

44. (4) Order of magnitude calculation is enough.

$$(2m_e eV)^{1/2} = (2 \times 9 \times 10^{-31} \times 100 \times 1.6 \times 10^{-19})^{1/2} \approx 5 \times 10^{-24} \text{ kg m s}^{-1}$$

$$\text{and } h \approx 6 \times 10^{-34} \text{ J s}$$

$$\text{So, } \lambda \approx 10^{-10} \text{ m}$$

Mid-wavelength in visible region is $\lambda_0 \approx 5000 \times 10^{-10} \text{ m}$

$$\text{Thus, } \lambda = \lambda_0 / 5000$$

$$45. (3) \quad 8 \times 10^{14} h = \phi_0 + 0.5$$

$$12 \times 10^{14} h = \phi_0 + 2$$

$$\text{Dividing, we get } \frac{12}{8} = \frac{\phi_0 + 2}{\phi_0 + 0.5}$$

$$\frac{3}{2} = \frac{\phi_0 + 2}{\phi_0 + 0.5}$$

$$3\phi_0 + 1.5 = 2\phi_0 + 4$$

$$\text{or } \phi_0 = 2.5 \text{ eV}$$

$$46. (1) \quad \lambda = \frac{h}{\sqrt{2mkT}} = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 1.38 \times 10^{-23} T}} \text{ m}$$

$$= \frac{6.62 \times 10^{-34}}{2.15 \times 10^{-25} \sqrt{T}} \text{ m}$$

$$= \frac{3.079}{\sqrt{T}} \times 10^{-9} \text{ m} = \frac{30.79}{\sqrt{T}} \text{ \AA} \approx \frac{30.8}{\sqrt{T}} \text{ \AA}$$

$$47. (4) \quad \lambda = \frac{h}{mv}$$

$$\text{Here, } 0 \times M = m_1 v_1 + m_2 v_2$$

$$\text{Clearly, } m_1 v_1 = -m_2 v_2$$

In magnitude, $mv = \text{constant}$

$$\therefore \frac{\lambda_1}{\lambda_2} = \frac{1}{1}$$

$$48. (2) \quad \text{Energy of electron in } n\text{th orbit, } E_n = -\frac{13.6}{n^2} \text{ eV}$$

$$\text{For first Bohr orbit, } n = 1; E_1 = -13.6 \text{ eV}$$

$$\text{For second Bohr orbit, } n = 2; E_2 = -\frac{13.6}{4} \text{ eV}$$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{E_2}{E_1}}$$

$$\text{or } \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{1}{4}} \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{1}{2}$$

49. (4) Each photon has associated with it an energy wave given by

$$E = hf = \frac{hc}{\lambda}$$

and graph of E vs λ is a hyperbola.

$$\text{Thus, } E \propto \frac{1}{\lambda}$$

$$50. (2) \quad \text{Energy corresponding to } 2000 \text{ \AA} = (12375/2000) \text{ eV} = 6.2 \text{ eV}$$

Maximum kinetic energy is $(6.2 - 5.01) \text{ eV} = 1.19 \text{ eV}$

$$\text{Now, } \frac{1}{2} \times 9.1 \times 10^{-31} \times v_{\text{max}}^2 = 1.19 \times 1.6 \times 10^{-19}$$

$$\text{or } v_{\text{max}}^2 = \frac{1.19 \times 1.6 \times 10^{-19} \times 2}{9.1 \times 10^{-31}}$$

$$= 0.418 \times 10^{12} = 41.8 \times 10^{10}$$

$$\text{or } v_{\text{max}} = 6.46 \times 10^5 \text{ m s}^{-1}$$

$$51. (3) \quad \lambda = \frac{h}{\sqrt{2mqV}}$$

$$mV = \text{constant}$$

$$1837 V' = 1 \times V$$

$$\text{or } V' = \frac{V}{1837} \text{ volt}$$

$$52. (3) E_k = \frac{12375}{4000} - \phi = 3.1 - \phi_0$$

$$2E_k = \frac{12375}{3100} - \phi = 3.99 - \phi_0$$

$$6.2 - 2\phi_0 = 3.99 - \phi_0$$

$$\text{or } \phi_0 = 6.2 - 3.99 = 2.21 \text{ eV}$$

$$53. (3) K_A = \frac{hc}{\lambda_A} - \phi_0; \quad K_B = \frac{hc}{\lambda_B} - \phi_0$$

$$\text{But } \lambda_A = 2\lambda_B, \text{ therefore}$$

$$\therefore K_A = \frac{hc}{2\lambda_B} - \phi_0$$

$$K_A = \frac{1}{2}[K_B + \phi_0] - \phi_0$$

$$\text{or } K_A = \frac{K_B}{2} - \frac{\phi_0}{2}$$

$$\therefore K_A < \frac{K_B}{2}$$

$$54. (2) \lambda = \frac{h}{\sqrt{2meV}}$$

$$\frac{hc}{\lambda_{\min}} = eV$$

$$\lambda \times \frac{hc}{\lambda_{\min}} = \frac{h}{\sqrt{2meV}} eV \quad \text{or} \quad \frac{\lambda}{\lambda_{\min}} \propto \sqrt{V}$$

$$55. (2) \frac{1}{2}mv^2 = hf - \phi_0$$

$$\frac{1}{2}mv'^2 = 4hf - \phi_0$$

$$\frac{v'^2}{v^2} = \frac{4hf - \phi_0}{hf - \phi_0}$$

$$\text{or } \frac{v'^2}{v^2} = \frac{4[hf - \phi] + 3\phi_0}{hf - \phi_0}$$

$$\text{Clearly, } v' > 2v$$

$$56. (1) \text{ Energy received by the eye, } E = \frac{nhc}{\lambda}$$

$$= \frac{5 \times 10^4 \times 6.6 \times 10^{-34} \times 3 \times 10^8}{5000 \times 10^{-10}} = 0.2 \times 10^{-13} \text{ W m}^{-2}$$

$$\text{So, eye is more sensitive by a factor of } \frac{1}{0.200} = 5.00$$

$$57. (3) E = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$

$$= \frac{(6.62 \times 10^{-34})^2}{2 \times 4 \times 1.67 \times 10^{-27} \times (0.1 \times 10^{-10})^2} \times \frac{1}{1.6 \times 10^{-19}} \text{ eV}$$

$$= \frac{43.82 \times 10^{-68}}{21.376 \times 10^{-68}} = 2.05 \text{ eV}$$

$$58. (2) \text{ The momentum of photon} = h/\lambda$$

If n is the number of photons falling per second on the plate, then total momentum per second of the incident photons is

$$p = n \times \frac{h}{\lambda}$$

Since the plate is blackened, all photons are absorbed by it.

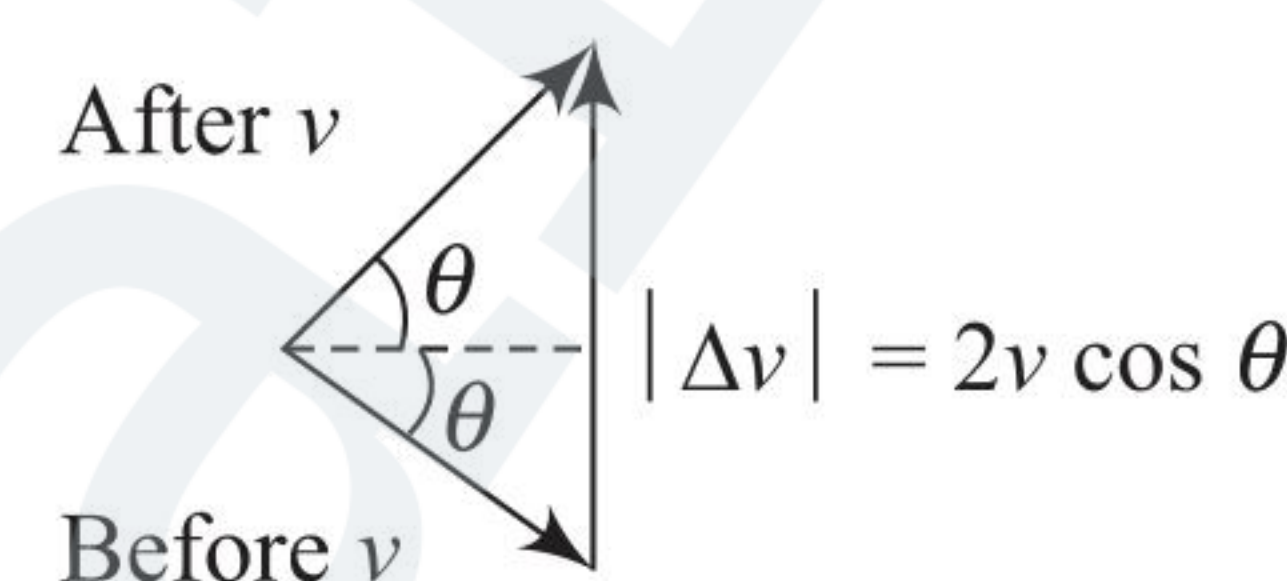
$$\frac{\Delta p}{\Delta t} = n \frac{h}{\lambda}$$

$$\text{Since } F = \frac{\Delta p}{\Delta t} = n \frac{h}{\lambda}$$

$$\therefore n = \frac{F\lambda}{h} = \frac{6.62 \times 10^{-5} \times 5 \times 10^{-7}}{6.62 \times 10^{-34}} = 5 \times 10^{22}$$

59. (1) Since the molecules rebound from the wall, the component of velocity perpendicular to the wall is reversed, while its velocity parallel to the wall does not change. The change in velocity of molecules is parallel to normal N . The magnitude of change is

$$|\Delta \vec{v}| = 2v \cos \theta$$



The change in momentum of a molecule is

$$|\Delta \vec{p}| = m |\Delta \vec{v}| = 2mv \cos \theta$$

in the direction of normal N . Let n be the number of molecules per unit volume. The number of molecules arriving at an area A of the wall per unit time is the number in a slanted cylinder whose length is equal to the velocity v and whose cross section is $A \cos \theta$.

$$\text{Number of molecules} = n (Av \cos \theta)$$

Each molecule suffers a change of momentum $2mv \cos \theta$.

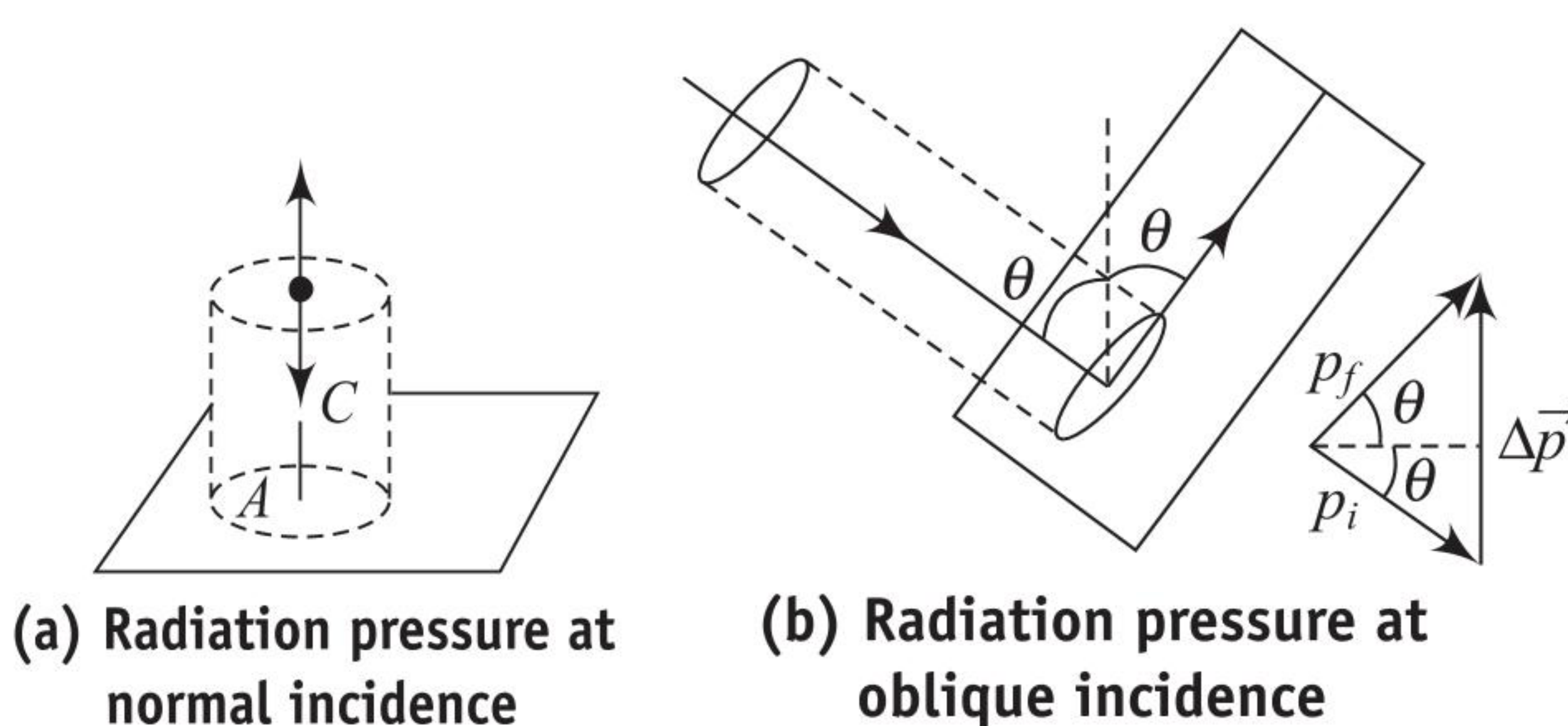
Change of momentum of a stream of gas in a direction perpendicular to the wall is equal to $(nAv \cos \theta) \times (2mv \cos \theta) = 2A nmv^2 \cos^2 \theta$.

Hence, force exerted on stream of gas by the wall,

$$F = 2Anmv^2 \cos^2 \theta$$

This is also the force exerted by gas molecules on the wall.

$$\text{Pressure} = \frac{\text{Normal force}}{\text{Area}} = \frac{F}{A} = 2nmv^2 \cos^2 \theta$$



(a) Radiation pressure at normal incidence

(b) Radiation pressure at oblique incidence

Remark: For oblique incidence, the change in momentum of the radiation per unit volume at the perfectly reflecting surface is $2p \cos \theta$ and the corresponding radiation pressure is

$$P_{\text{rad}} = 2pc \cos^2 \theta = 2E \cos^2 \theta$$

60. (4) Momentum corresponding to incident protons normal to the surface,

$$\left(\frac{dp}{dt} \right)_{\text{incident}} = \frac{I}{c} dA \cos^2 \theta$$

Since reflection coefficient is ρ , so the momentum of the reflected photons per second normal to surface,

$$\left(\frac{dp}{dt} \right)_{\text{reflected}} = -\frac{I}{c} dA \rho \cos^2 \theta$$

Hence, rate of change of momentum of the photons,

$$\left(\frac{dp}{dt}\right)_{\text{photons}} = -\frac{I}{c} dA(\rho + 1) \cos^2 \theta$$

From Newton's third law, $\left(\frac{dp}{dt}\right)_{\text{surface}} = -\frac{I}{c} dA(\rho + 1) \cos^2 \theta$

Hence, pressure exerted on surface,

$$P = \frac{dF}{dA} = \frac{I}{c} dA(\rho + 1) \cos^2 \theta$$

On substituting values, we get $P = 0.5 \text{ N cm}^{-2}$.

61. (1) Imagine the sphere to be made of thin circular rings of radius r , thickness $ds = R d\theta$ and subtending an angle of θ at the centre.

Momentum per second of incident photons,

$$\left(\frac{dp}{dt}\right)_{\text{incident}} = \frac{I}{c} dA \cos^2 \theta$$

Since surface of mirror is considered to be ideal, i.e., reflection coefficient is unity, photons suffer momentum change in normal direction only.

$$\left(\frac{dp}{dt}\right)_{\text{photon}} = \frac{2I}{c} dA \cos^2 \theta$$

$$dF_n = \left(\frac{dp}{dt}\right)_{\text{ball}} = +\frac{2I}{c} dA \cos^2 \theta$$

This force may be resolved into horizontal and vertical components. The vertical component $dF_n \sin \theta$ is cancelled because every element on the upper half has a symmetrically placed element in the lower half. So, resultant force on the ball,

$$F = \int dF_n \cos \theta = \int \frac{2I}{c} dA \cos^3 \theta$$

$$dA = (2\pi R \sin \theta) R d\theta$$

$$F = \int_0^{\pi/2} 4\pi \frac{I}{c} R^2 \cos^3 \theta \sin \theta d\theta$$

$$= \frac{4\pi R^2 I}{c} \int_0^{\pi/2} \cos^3 \theta \sin \theta d\theta = \frac{\pi R^2 I}{c}$$

On substituting values, we get $F = 0.8 \mu\text{N}$.

62. (3) Photons have momentum ($p = h/\lambda$) which they carry away; the spacecraft will acquire momentum in the opposite direction according to law of conservation of momentum.

Number of photons per second from laser = n

Then, from energy considerations, $0.5 \times 10^{-3} = nh \left(\frac{c}{\lambda}\right)$
 $n = (0.5 \times 10^{-3}) \lambda / (ch)$

Rate of change of momentum of spacecraft

$$= np = n \frac{h}{\lambda} = (0.5 \times 10^{-3}) \frac{\lambda}{ch} \left(\frac{h}{\lambda}\right) = \frac{0.5 \times 10^{-3}}{c}$$

From Newton's second law, $\frac{nh}{\lambda} = ma$

$$1000a = \frac{0.5 \times 10^{-3}}{3.00 \times 10^8} = \frac{1}{6} \times 10^{-11}$$

$$v = at$$

$$t = \frac{v}{a} = \frac{1000}{\left(\frac{1}{1000}\right) \times \frac{1}{6} \times 10^{-11}} \text{ s} = 6 \times 10^{17} \text{ s}$$

63. (2) The gain of kinetic energy by an electron is eV .

$$\frac{1}{2} mv^2 = eV$$

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2(1.60 \times 10^{-19})(50)}{(9.11 \times 10^{-31})}} = 4.19 \times 10^6 \text{ m s}^{-1}$$

Thus, the electron's de Broglie wavelength is

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{(9.11 \times 10^{-31})(4.19 \times 10^6)} = 1.74 \times 10^{-10} \text{ m}$$

64. (4) Magnetic force experienced by a charged particle in a magnetic field is given by

$$F_B = q\vec{v} \times \vec{B} = qvB \sin \theta$$

In our case, $F_B = qvB$ [as $\theta = 90^\circ$]

$$\text{Hence, } Bqv = \frac{mv^2}{r} \Rightarrow mv = qBr$$

The de Broglie wavelength, $\lambda = \frac{h}{mv} = \frac{h}{qBr}$

$$\frac{\lambda_{\alpha\text{-particle}}}{\lambda_{\text{proton}}} = \frac{q_p r_p}{q_\alpha r_\alpha}$$

$$\text{Since } \frac{r_\alpha}{r_p} = 1 \text{ and } \frac{q_\alpha}{q_p} = 2$$

$$\Rightarrow \frac{\lambda_\alpha}{\lambda_p} = \frac{1}{2}$$

65. (1) Kinetic energy gained by a charge q after being accelerated through a potential difference V volt.

$$qV = \frac{1}{2} mv^2$$

$$v = \sqrt{\frac{2qV}{m}}$$

$$mv = \sqrt{2mqV}$$

de Broglie wavelength = $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mqV}}$

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha V_\alpha}{m_p q_p V_p}}$$

$$\text{Putting } V_\alpha = V_p, \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{4 \times 2}{1 \times 1}} = 2\sqrt{2}$$

66. (4) The electrons ejected with maximum speed v_{max} are stopped by electric field $E = 4 \text{ N/C}$ after traveling a distance $d = 1 \text{ m}$.

$$\frac{1}{2} mv_{\text{max}}^2 = eEd = 4 \text{ eV}$$

The energy of incident photon = $\frac{1240}{200} = 6.2 \text{ eV}$

From equation of photoelectric effect $\frac{1}{2} mv_{\text{max}}^2 = h\nu - \phi_0$

$$\therefore \phi_0 = 6.2 - 4 = 2.2 \text{ eV}$$

67. (1) We have $\text{KE} = \frac{p^2}{2m_e} = \frac{hc}{\lambda_{\text{min}}}$

$$p = \sqrt{\frac{2hcm_e}{\lambda_{\text{min}}}}$$

$$\text{Also, } \lambda_{\text{de Broglie}} = \frac{h}{p} = \sqrt{\frac{h\lambda_{\text{min}}}{2m_e c}}$$

For $\lambda_{\text{min}} = 10 \text{ \AA}$,

$$\lambda_{\text{de Broglie}} \cong 0.3 \text{ \AA}$$

68. (3) $K_{\text{max}} = hf - \phi$

$$\Rightarrow \frac{1}{2} mv_{\text{max}}^2 = hf - \phi$$

$$\Rightarrow v_{\max} = \sqrt{\frac{2(hf - \phi)}{m}}$$

Hence, (1) is incorrect.

Since $n = (IA/hf)$, therefore rate of emission of electrons is proportional to the intensity (I).

$$K_{\max} = hf - \phi$$

Hence, (3) is true.

69. (2) The intensity of light at the location of your eye is

$$I = \frac{P}{4\pi r^2} = \frac{60}{4\pi \times 4^2} \text{ W m}^{-2}$$

The energy entering into your eye per second is

$$P_1 = I \times \frac{\pi d^2}{4}$$

where d is the diameter of pupil.

$$P_1 = \frac{60}{4\pi \times 4^2} \times \frac{\pi \times (2 \times 10^{-3})^2}{4} = 9.375 \times 10^{-7} \text{ J s}^{-1}$$

Let n be the number of photons entering into the eye per second, then

$$P_1 = n \times \frac{hc}{\lambda}$$

$$9.375 \times 10^{-7} = n \times \frac{1240 \times 1.6 \times 10^{-19}}{600}$$

$$n = 2.84 \times 10^{12} \text{ photon s}^{-1}$$

So, the number of photons entering the eye in 0.1 s = 0.1 n
 $= 2.84 \times 10^{11}$

70. (2) Let m be the mass of each particle, then $\lambda_1 = (h/mv_1)$ and $\lambda_2 = (h/mv_2)$, where v_1 and v_2 are the velocities of two particles as shown in the figure.

$$\vec{v}_{CM} = \frac{m\vec{v}_1 + m\vec{v}_2}{2m} = \frac{\vec{v}_1 + \vec{v}_2}{2}$$

Velocity of A w.r.t. C frame is

$$\vec{v}_{1c} = \vec{v}_1 - \vec{v}_{CM} = \frac{\vec{v}_1 - \vec{v}_2}{2}$$

$$|\vec{v}_{1c}| = \frac{\sqrt{v_1^2 - v_2^2}}{2} = |\vec{v}_{2c}|$$

So, required wavelength is

$$\lambda = \frac{h}{m|\vec{v}_{1c}|} = \frac{h}{m} \times \frac{2}{\frac{h}{m} \sqrt{\frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}}} = \frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}$$

71. (4) $K_{\max} = hf - \phi$

$$= \left(\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{0.3 \times 10^{-6} \times 1.6 \times 10^{-19}} - 2.46 \right) \text{ eV} = 1.68 \text{ eV}$$

$$\text{Cut-off wavelength, } \lambda_0 = \frac{hc}{\phi} = 505 \text{ nm}$$

The minimum energy required to eject the photo-electrons is equal to work function.

72. (1) Let m be the mass of particle.

$$\frac{mv^2}{2} = \frac{hc}{\lambda_{\text{photon}}}, \text{ where symbols have their usual meanings.}$$

$$\frac{p^2}{2m} = \frac{hc}{\lambda_{\text{photon}}}$$

$$\text{and } p = \frac{h}{\lambda_{\text{particle}}} \Rightarrow \frac{h^2}{2m\lambda_{\text{particle}}^2} = \frac{hc}{\lambda_{\text{photon}}}$$

$$\Rightarrow \frac{\lambda_{\text{photon}}}{\lambda_{\text{particle}}} = \frac{2mc}{h} \times \lambda_{\text{particle}} = \frac{2mc}{h} \times \frac{h}{mv} = \frac{2c}{0.05c} = 40$$

73. (2) Resolving power is proportional to inverse of wavelength,

$$\text{i.e., } R \propto \frac{1}{\lambda}$$

$$\text{and } \lambda \propto \frac{1}{p}$$

$$\text{So, } R \propto p = \sqrt{2mE}$$

$$\text{So, } R \propto \sqrt{E}$$

$$\frac{R'}{R} = \sqrt{\frac{4\text{ kV}}{16\text{ kV}}} = \frac{1}{2}$$

$$R' = \frac{R}{2}$$

74. (4) Speed of electron which enters into electric field may increase or decrease while for second electron, it remains constant.

$$\text{So, from } \lambda = \frac{h}{mv'}$$

$$\lambda_1 > \lambda_2 \text{ or } \lambda_1 < \lambda_2$$

$$75. (2) \lambda_1 = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

$$\lambda_2 = \frac{hc}{E}$$

$$\text{So, } \frac{\lambda_1}{\lambda_2} \propto \frac{E}{\sqrt{E}} = E^{1/2}$$

76. (1) When the positive potential of the ball is enough to hold back the most energetic photoelectron, the ball will not emit photoelectrons.

$$\frac{hc}{\lambda} - \phi = Ve$$

$$V = \frac{\frac{hc}{\lambda} - \phi}{e}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{200 \times 10^{-9} \times 1.6 \times 10^{-19}} - 3.74$$

$$= 6.216 - 3.74 = 2.5 \text{ V}$$

$$77. (1) E = \frac{hc}{\lambda}$$

$$\text{Number of photons emitted is } \frac{Pt}{\left(\frac{hc}{\lambda}\right)} = n_0$$

$$n_0 = \frac{P\lambda t}{hc}$$

Since the radiation is spherically symmetric, so total number of photons entering the sensor is n_0 times the ratio of aperture area to the area of a sphere of radius l .

$$N = n_0 \frac{\pi(2d)^2}{4\pi l^2} = \frac{P\lambda t d^2}{hc l^2}$$

$$78. (2) \lambda_{\text{ph}} = \lambda_e$$

$$\frac{h}{p_{\text{ph}}} = \frac{h}{p_e}$$

$$\frac{E_{\text{ph}}}{c} = \frac{2E_e}{v}$$

$$\frac{E_e}{E_{\text{ph}}} = \frac{v}{2c}$$

79. (4) From conservation of linear momentum, both the particles will have equal and opposite momentum. The de-Broglie wavelength is given by

$$\lambda = \frac{h}{p} \Rightarrow \lambda_1/\lambda_2 = 1$$

80. (1) $mvr = n \frac{h}{2\pi} = 2 \left(\frac{h}{2\pi} \right) \quad (n = 2)$

or $mvr = \frac{h}{\pi}$

$\therefore \lambda = \frac{h}{mv} = \text{de Broglie wavelength}$
 $= \pi r = (3.14) (2.116 \text{ \AA}) = 6.64 \text{ \AA}$

81. (2) $h(f_1 - f_2) = e(V_1 - V_2)$

$$\frac{h}{e} = \frac{1}{c} \left(\frac{V_1 - V_2}{\frac{1}{\lambda_1} - \frac{1}{\lambda_2}} \right) = \frac{10^{-9}}{3 \times 10^8} \left(\frac{4.6 - 0.08}{\frac{1}{185} - \frac{1}{546}} \right)$$

$$= \frac{4.42 \times 185 \times 546 \times 10^{-17}}{361 \times 31}$$

$$= 4.12 \times 10^{-15} \text{ Js C}^{-1}$$

82. (4) Momentum of photon of green light,

$$p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}}{505 \times 10^{-9}} = 1.3 \times 10^{-27} \text{ kg ms}^{-1}$$

Velocity of bacterium, $v = \frac{p}{m} = \frac{1.3 \times 10^{-27}}{9.5 \times 10^{-15}} = 1.368 \times 10^{-13} \text{ ms}^{-1}$

83. (1) Energy of dissociation, $E_s = 10^5 \text{ J mol}^{-1}$

Photon energy, $E_p = \frac{E_s}{N_a} = \frac{10^5}{6.02 \times 10^{23}} = 1.66 \times 10^{-19} \text{ J} = 1.04 \text{ eV}$

Multiple Correct Answers Type

1. (1), (2), (3)

Existence of cut-off frequency and photoemission takes place even when intensity is low.

2. (2), (4)

$$eV_0 = K_{\max} = \frac{hc}{\lambda} - W$$

3. (1), (3)

Use Einstein's photoelectric equation.

4. (1), (2)

For photoemission, $\lambda < \lambda_0$.

5. (1), (3), (4)

$$E = \frac{1240}{248} \text{ eV}$$

$$E_1 = 5 - 2.2 = 2.8 \text{ eV}$$

$$E_2 = 5 \times 0.8 - 2.2 = 1.8 \text{ eV}$$

$$E_3 = 5 \times 0.8^2 - 2.2 = 1 \text{ eV}$$

$$E_4 = 5 \times 0.8^3 - 2.5 = 0.36 \text{ eV}$$

6. (1), (2), (3), (4)

Stopping potential $\propto$ frequency $\equiv \frac{1}{\text{wavelength}}$

Saturation current $\propto$ rate of photoelectron emission.

Also, $K_{\max} = h\nu - \phi$, $p = \sqrt{2mK}$

7. (3), (4)

The threshold wavelength is 5200 Å. For ejection of electrons, the wavelength of the light should be less than 5200 Å so that frequency increases and hence the energy of incident photon increases. UV light has less wavelength than 5200 Å.

8. (2), (4)

Since the stopping potential depends on the frequency and not on the intensity and the source is same, the stopping potential remains unaffected. The saturation current depends on the intensity of incident light on the cathode of the photocell which in turn depends on the distance of the source from cathode. The intensity of light is inversely proportional to the square of the distance between the light source and photocell.

Intensity, $I \propto 1/r^2$ and saturation current $\propto I$ (Intensity)

$$\Rightarrow \text{Saturation current} \propto \frac{1}{r^2}$$

$$\Rightarrow \frac{(\text{Saturation current})_{\text{final}}}{(\text{Saturation current})_{\text{initial}}} = \frac{r_{\text{initial}}^2}{r_{\text{final}}^2}$$

$$\Rightarrow (\text{Saturation current})_{\text{final}} = \frac{0.2 \times 0.2}{0.6 \times 0.6} \times 18 = 2 \text{ mA}$$

9. (1), (2), (3)

For metal A: $4.25 = W_A + T_A$... (i)

Also, $T_A = \frac{1}{2}mv_A^2 = \frac{1}{2} \frac{m^2 v_A^2}{m} = \frac{p_A^2}{2m} = \frac{h^2}{2m\lambda_A^2}$... (ii)

$$\left[\because \lambda = \frac{h}{p} \right]$$

For metal B: $4.7 = (T_A - 1.5) + W_B$... (iii)

Also, $T_B = \frac{h^2}{2m\lambda_B^2} \times \frac{2m\lambda_A^2}{h^2} = \frac{\lambda_A^2}{\lambda_B^2}$

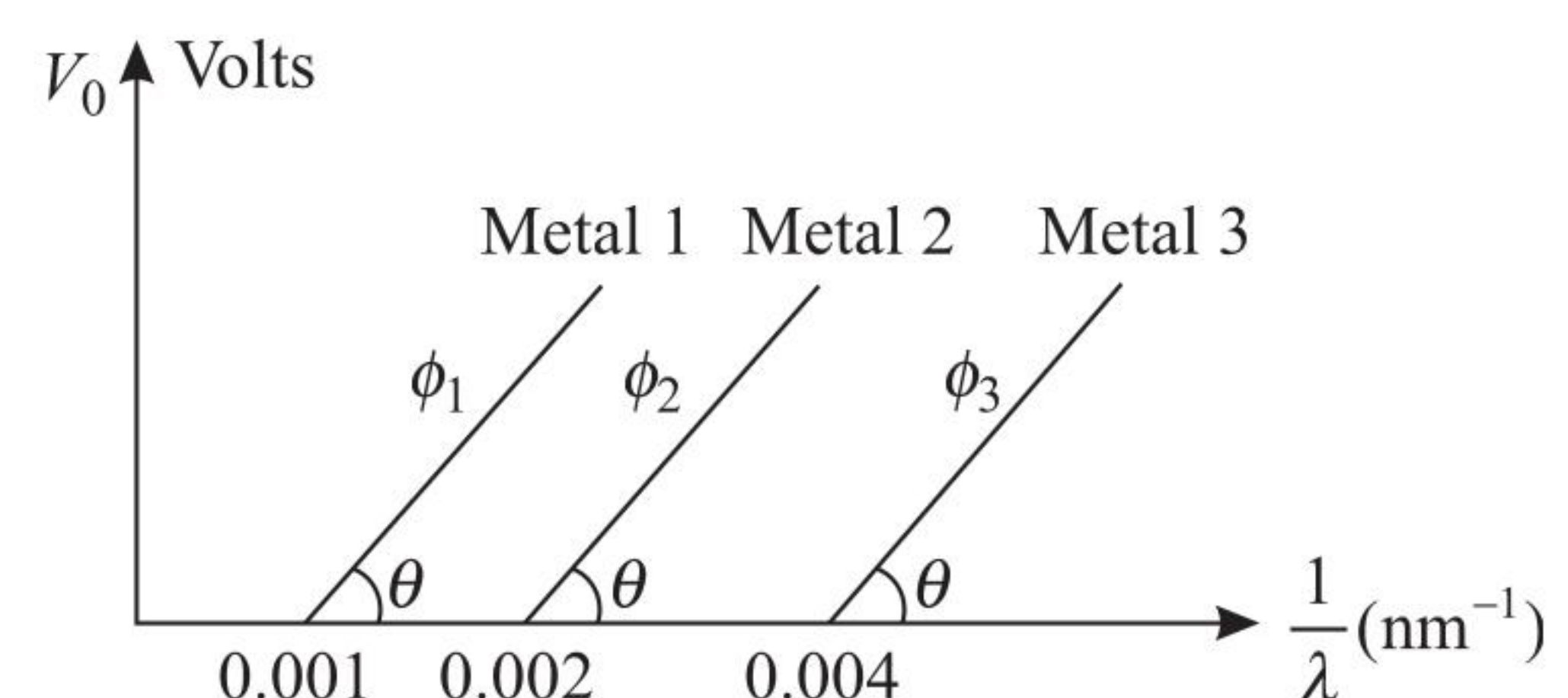
$$\Rightarrow \frac{T_A - 1.5}{T_A} = \frac{\lambda_A^2}{(2\lambda_A)^2} = \frac{\lambda_A^2}{4\lambda_A^2} = \frac{1}{4} \quad [\because \lambda_B = 2\lambda_A \text{ given}]$$

$$\Rightarrow 4T_A - 6 = T_A \Rightarrow T_A = 2 \text{ eV}$$

$$\therefore W_A = 2.25 \text{ eV}, W_B = 4.20 \text{ eV}, T_B = 3 \text{ eV}$$

10. (1), (3)

$$\phi_1 : \phi_2 : \phi_3 = eV_{01} : eV_{02} : eV_{03}$$



$$V_{01} : V_{02} : V_{03} = 0.001 : 0.002 : 0.004 = 1 : 2 : 4$$

Therefore, option (1) is correct.

By Einstein's photoelectric equation, $\frac{hc}{\lambda} - \phi = eV$

$$\Rightarrow V = \frac{hc}{e\lambda} - \frac{\phi}{e} \quad \dots (i)$$

Comparing Eq. (i) by $y = mx + c$, we get the slope of the line

$$m = \frac{hc}{e} = \tan \theta$$

$\Rightarrow$ Option (3) is correct.

From the graph it is clear that, $\frac{1}{\lambda_{01}} = 0.001 \text{ nm}^{-1}$

$$\Rightarrow \lambda_{01} = \frac{1}{0.001} = 1000 \text{ nm}$$

Also, $\frac{1}{\lambda_{02}} = 0.002 \text{ nm}^{-1} \Rightarrow \lambda_{02} = 500 \text{ nm}$

and $\lambda_3 = 250 \text{ nm}$

Violet colour light will have wavelength less than 400 nm. Therefore, this light will be unable to show photoelectric effect on plate.

11. (1), (3), (4)

(a) For the first wavelength: $eV_{s1} = hf - W$... (i)

W = work function

For the second surface: $eV_{s2} = hf_2 - W$... (ii)

Subtracting, we get $V_{s2} - V_{s1} = \frac{h}{e}(f_2 - f_1)$

$$\begin{aligned} \text{or } V_{s2} &= V_{s1} + \frac{hc}{e} \left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right) = V_{s1} + \frac{hc}{e} \left(\frac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2} \right) \\ &= 0.19 + 1240 \left(\frac{550 - 190}{190 \times 550} \right) = 4.47 \text{ V} \end{aligned}$$

(b) From Eq. (i): $W = \frac{hc}{\lambda_1} - eV_{s1}$

Work function (in eV),

$$\frac{W}{e} = \frac{hc}{e\lambda_1} - V_{s1} = \frac{1240}{550} - 0.19 = 2.07 \text{ eV}$$

(c) Threshold frequency,

$$f_0 = \frac{W}{h} = \frac{2.07 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}} = 5 \times 10^{14} \text{ Hz}$$

12. (1), (3), (4)

Let N be the number of photoelectrons per second.

$\therefore$ Intensity = $N \times$ energy of one photon $\Rightarrow I = N \frac{hc}{\lambda}$

$\therefore$ Number of incident photons,

$$\begin{aligned} N_i &= \text{Incident intensity} \times \frac{\lambda}{hc} \\ &= \frac{10^{-8} \times 365 \times 10^{-9}}{6.62 \times 10^{-34} \times 3 \times 10^8} = 18.35 \times 10^9 \end{aligned}$$

Number of photon N_{ab} absorbed by the surface per unit area per unit time (i.e., absorbed flux),

$$N_{ab} = 0.8 \times 18.35 \times 10^9 = 1.47 \times 10^{10} \text{ m}^{-2} \text{ s}^{-1}$$

Emitted flux = No. of electrons emitted $\text{m}^{-2} \text{s}^{-1} = 1.47 \times 10^{10} \text{ m}^{-2} \text{s}^{-1}$

Power absorbed per $\text{m}^2 = 0.8 \times$ Incident intensity

$$= 0.8 \times 10^{-8} = 8 \times 10^{-9} \text{ W m}^{-2}$$

From Einstein's photoelectric equation,

$$\begin{aligned} K_{\max} &= hf - W_0 = \frac{hc}{\lambda} - W_0 \\ &= \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{365 \times 10^{-9}} - 1.6 \times 1.6 \times 10^{-19} \\ &= 2.89 \times 10^{-19} \text{ J} = 1.80 \text{ eV} \end{aligned}$$

13. (1), (3)

(a) The energy of each photon is

$$\begin{aligned} E &= \frac{hc}{\lambda} = \frac{(4.14 \times 10^{-15} \text{ eV-s}) \times (3 \times 10^8 \text{ m/s})}{500 \text{ nm}} \\ &= \frac{1242 \text{ eV-nm}}{500 \text{ nm}} = 2.48 \text{ eV} \end{aligned}$$

In 1 s, 10 J of energy passes through any cross section of the beam. Thus, the number of photons crossing a cross section per unit time is

$$n = \frac{10 \text{ J}}{2.48 \text{ eV}} = 2.52 \times 10^{19}$$

This is also the number of photons falling on the surface per second and being absorbed.

(b) The linear momentum of each photon is $p = h/\lambda = hf/c$

The total momentum of all the photons falling per second on the surface is

$$\frac{nhf}{c} = \frac{10 \text{ J}}{3 \times 10^8 \text{ m/s}} = 3.33 \times 10^{-8} \text{ N-s}$$

As the photons are completely absorbed by the surface, this much momentum is transferred to the surface per second. The rate of change of the momentum of the surface, i.e., the force on it is

$$F = \frac{dp}{dt} = \frac{3.33 \times 10^{-8} \text{ N-s}}{1 \text{ s}} = 3.33 \times 10^{-8} \text{ N}$$

Linked Comprehension Type

For Problems 1–3

1. (2) $\lambda_0 = 3800 \text{ Å}$

$$\begin{aligned} W &= hf_0 = h \frac{c}{\lambda_0} = \frac{6.633 \times 10^{-34} \times 3 \times 10^8}{3800 \times 10^{-10}} \\ &= 5.23 \times 10^{-19} \text{ J} = 3.27 \text{ eV} \end{aligned}$$

2. (1) Incident wavelength $\lambda = 2600 \text{ Å}$

$$f = \text{incident frequency} = \frac{3 \times 10^8}{2600 \times 10^{-10}} \text{ Hz}$$

Then, $K_{\max} = hf - W_0$

$$hf = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2600 \times 10^{-10}} = 7.65 \times 10^{-19} \text{ J} = 4.78 \text{ eV}$$

$$K_{\max} = hf - W_0 = 4.78 \text{ eV} - 3.27 \text{ eV} = 1.51 \text{ eV}$$

3. (1) $K_{\max} = \frac{1}{2} m v_{\max}^2$

$$\text{or } v_{\max} = \sqrt{\frac{2K_{\max}}{m}} = 72.89 \times 10^8 \text{ m s}^{-1}$$

For Problems 4–6

4. (4) Energy of a photon of wavelength λ ,

$$E = hf = (hc/\lambda)$$

$$E = \frac{(6.6 \times 10^{-34}) \times (3 \times 10^8)}{6000 \times 10^{-10}} = 3.3 \times 10^{-19} \text{ J}$$

So, if n is the number of photons emitted per second,

$$nE = \frac{\text{energy}}{\text{second}} = \text{power (P)}$$

$$\text{Hence, } n = \frac{P}{E} = \frac{100}{3.3 \times 10^{-19}} \approx 3 \times 10^{20}, \text{ photons s}^{-1}$$

5. (2) Intensity (i.e., energy flux) at a distance of 5 m from the source of power P ,

$$I = \frac{P}{4\pi r^2} = \frac{100}{4\pi \times 5^2} = \frac{1}{\pi} \text{ Wm}^{-2}$$

The number of photons passing normally per unit area per second, i.e.,

$$\text{Photon flux} = \frac{I}{E} = \frac{1/\pi}{3.3 \times 10^{-19}} \approx 10^{18} \text{ photons m}^{-2} \text{ s}^{-1}$$

6. (3) Energy of photon

$$E = hf = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{4000 \times 10^{-10}} \approx 5 \times 10^{-19} \text{ J}$$

So, number of photons emitted per second by light source,

$$n_p = \frac{P}{E} = \frac{1.5 \times 10^{-3}}{5 \times 10^{-19}} = 3 \times 10^{15}$$

Now, as only 0.1% of the photons emit electrons, number of photoelectrons emitted per second,

$$n_e = \frac{0.1}{100} \times 3 \times 10^{15} = 3 \times 10^{12}$$

But current is the rate of flow of charge, i.e.,

$$I = \frac{q}{t} = \frac{Ne}{t} = n_e \times e \quad \left[\text{as } \frac{N}{t} = n_e \right]$$

$$\text{So, } I = 3 \times 10^{12} \times 1.6 \times 10^{-19} = 0.48 \text{ } \mu\text{A}$$

For Problems 7–8

7. (3) From Einstein's photoelectric effect equation, $\frac{hc}{\lambda} = \phi_0 + \frac{1}{2}mv^2$

$$\text{For } \lambda_1 = 3000 \text{ } \text{\AA}, \quad \frac{hc}{3000 \times 10^{-10}} = \phi_0 + \frac{1}{2}m(3v)^2 \quad \dots(i)$$

$$\text{For } \lambda_1 = 6000 \text{ } \text{\AA}, \quad \frac{hc}{6000 \times 10^{-10}} = \phi_0 + \frac{1}{2}mv^2 \quad \dots(ii)$$

Now, on multiplying Eq. (ii) by 9 and subtracting Eq. (i) from it, we get

$$8\phi_0 = 9 \frac{hc}{6000 \times 10^{-10}} - \frac{hc}{3000 \times 10^{-10}}$$

Substituting the values, we get $\phi_0 = 1.23 \text{ eV}$

8. (1) Maximum speed of the photoelectrons will be for the incident light of wavelength $\lambda = 3000 \text{ } \text{\AA}$. From Eq. (i)

$$\frac{1}{2}m(3v)^2 = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{3000 \times 10^{-10}} - 2.896 \times 10^{-19}$$

$$= 3.724 \times 10^{-19}$$

$$\therefore 3v = v_{\max} = \left(\frac{2 \times 3.724 \times 10^{-19}}{9.1 \times 10^{-31}} \right)^{1/2} = 9 \times 10^5 \text{ m s}^{-1}$$

For Problems 9–10

9. (2) The energy of each photon is $E = h\nu$

Since the wavelength and frequency are related to the speed of light by

$$c = \nu\lambda$$

$$E = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34})(3.00 \times 10^8)}{632.8 \times 10^{-9}} = 3.14 \times 10^{-19} \text{ J}$$

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

$$E = \frac{3.14 \times 10^{-19}}{1.602 \times 10^{-19}} = 1.96 \text{ eV}$$

10. (1) The number of photons emitted per second is equal to the energy emitted by the laser each second divided by the energy of one photon.

$$N = \frac{1.00 \times 10^{-3}}{3.14 \times 10^{-19}} = 3.18 \times 10^{15} \text{ photon s}^{-1}$$

For Problems 11–13

11. (1) From Einstein's relation, $eV_s = hf - W$

As work function is a constant for a surface,

$$e(V_{s_2} - V_{s_1}) = h(f_2 - f_1)$$

$$V_{s_2} = V_{s_1} + \frac{h}{e}(f_2 - f_1) = V_{s_1} + \frac{hc}{e} \left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right)$$

$$= 0.19 + 1240 \left(\frac{1}{190} - \frac{1}{550} \right) = 4.47 \text{ V}$$

$$\mathbf{12. (2) } W = \frac{hc}{\lambda_1} - eV_{s_1} = \frac{1240}{550} - 0.19 = 2.07 \text{ eV}$$

13. (1) $hf_c = W$

$$f_c = \frac{W}{h} = \frac{(2.07)(1.602 \times 10^{-19})}{6.626 \times 10^{-34}} \approx 500 \times 10^{12} \text{ Hz}$$

For Problems 14–15

$$\mathbf{14. (4) } \text{Energy of photon, } E = \frac{hc}{\lambda} = \frac{1.24 \times 10^3}{400} = 3.1 \text{ eV}$$

$$\text{Remaining energy} = 3.1 - 0.31 = 2.79 \text{ eV}$$

$$\text{Energy lost in first collision is } (3.1) \times \left(\frac{10}{100} \right) = 0.31 \text{ eV}$$

$$\text{Remaining energy is } 3.1 - 0.31 = 2.79 \text{ eV}$$

$$\text{Energy lost in second collision is } (2.79) \times \left(\frac{10}{100} \right) = 0.279 \text{ eV}$$

$$\text{Total energy lost in two collisions is } (0.31) + (0.279) \text{ eV} = 0.589 \text{ eV}$$

So, from conservation of energy, we have

$$\frac{hc}{\lambda} = \phi + K_{\max} + \text{energy lost in two collision}$$

$$3.1 = 2.2 + K_{\max} + 0.589$$

$$K_{\max} = 0.31 \text{ eV}$$

$$\text{Total energy after second collision is } (2.79 - 0.279) = 2.511 \text{ eV}$$

$$\mathbf{15. (3) } \text{Energy lost in third collision is } 2.511 \times \frac{10}{100} = 0.2511 \text{ eV}$$

$$\text{Remaining energy} = (2.511 - 0.2511) = 2.2599 \text{ eV}$$

$$\text{Energy lost in fourth collision} = \left(2.2599 \times \frac{10}{100} \right) = 0.2259 \text{ eV}$$

$$\text{Remaining energy} = (2.2599 - 0.2259) = 2.034 \text{ eV}$$

After the fourth collision, the electron does not have enough energy to overcome the work function, so it cannot come out.

For Problems 16–18

16. (4) If P is the power of point source of light, the intensity at a distance r is

$$I = \frac{P}{4\pi r^2}$$

The energy intercepted by the metallic sphere is

$$E = \text{intensity} \times \text{projected area of sphere} = \frac{P}{4\pi r^2} \times \pi R^2$$

If ϵ is the energy of the single photon and η the efficiency of the photon to liberate an electron, the number of ejected electrons is

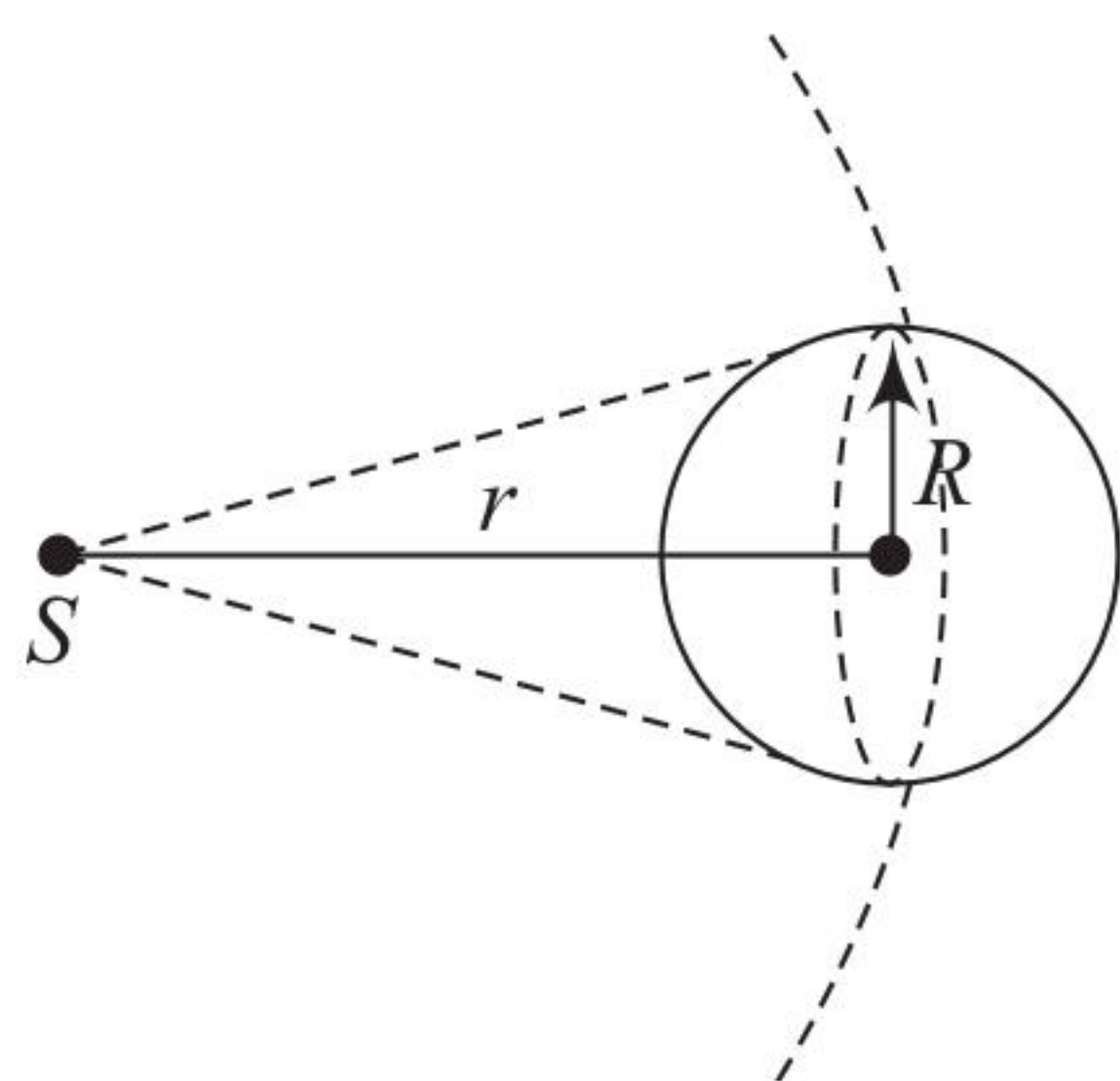
$$\eta \frac{PR^2}{4r^2\epsilon} = \frac{(10^{-6})(3.2 \times 10^{-3})(8 \times 10^{-3})^2}{4 \times (0.8)^2 \times (5 \times 1.6 \times 10^{-19})} = 10^5 \text{ electron s}^{-1}$$

17. (2) The emission of electrons from a metallic sphere leaves it positively charged. As the potential of the charged sphere begins to rise, it attracts emitted electrons. The emission of electrons will stop when the kinetic energy of the electrons is neutralised by the retarding potential of the sphere. So, we have

$$eV = K_{\max}$$

$$V = \left(\frac{K_{\max}}{e} \right)$$

18. (1)



From Einstein's photoelectric equation,

$$K_{\max} = h\nu - \phi = (5 - 3) = 2 \text{ eV}$$

The potential of a charged sphere is $V = \frac{1}{4\pi\epsilon_0} \frac{q}{R} = \frac{1}{4\pi\epsilon_0} \left(\frac{ne}{R} \right)$

$$\frac{1}{4\pi\epsilon_0} \left(\frac{ne}{R} \right) = 2$$

$$n = \frac{4\pi\epsilon_0 2R}{e} = \frac{2 \times 8 \times 10^{-3}}{9 \times 10^9 \times 1.6 \times 10^{-19}} = 1.11 \times 10^7$$

The photoelectric effect will stop when 1.11×10^7 electrons have been emitted.

The time taken by it to emit 1.11×10^7 electrons,

$$t = \frac{1.11 \times 10^7}{10^5} = 111 \text{ s} = 1.85 \text{ min}$$

For Problems 19–20

19. (2) Pressure exerted by absorbed light = $\frac{1}{2} \left(\frac{S}{c} \right)$

Pressure exerted by reflected light = $\frac{1}{2} \left(\frac{2S}{c} \right)$

Total radiation pressure on the surface is

$$P_{\text{rad}} = \frac{\frac{3}{2}S}{c} = \frac{1.5 \times 10^3}{3 \times 10^8} = 5 \times 10^{-6} \text{ Pa}$$

20. (1) $\frac{P_{\text{rad}}}{P_0} = \frac{5 \times 10^{-6}}{1 \times 10^5} = 5 \times 10^{-11}$

For Problems 21–25

21. (3) $Q = CV \Rightarrow ne = \frac{\epsilon_0 A}{d} V$

$$n = \frac{2.85 \times 10^{-12} \times 10}{0.5 \times 10^{-3} \times 1.6 \times 10^{-19}} \times 16$$

$$n = 8.85 \times 10^9$$

22. (3) Equivalent resistance $R = \frac{V}{I} = \frac{16 \text{ V}}{2 \times 10^{-6} \text{ A}} = 8 \times 10^6 \Omega$

23. (4) $P = \frac{nhc}{e\lambda}$

where n = number of photons incident per unit time.

Also, $I = ne$

$$\Rightarrow P = \frac{Ihc}{e\lambda}$$

$$\lambda = \frac{(2 \times 10^{-6})(6.6 \times 10^{-34})(3 \times 10^8)}{(4 \times 10^{-6})(1.6 \times 10^{-19})}$$

$$= \frac{9.9}{1.6} \times 10^{-7} \text{ m} = \frac{9900}{1.6} \text{ \AA} = 6187 \text{ \AA}$$

which is in the range of orange light.

24. (3) The range of wavelength for red light is beyond the wavelength of incident light.

25. (2) Stopping potential, $V_s = 8 \text{ V}$

and $\text{KE} = eV_s$

$\therefore \text{KE} = 8 \text{ eV}$

For Problems 26–28

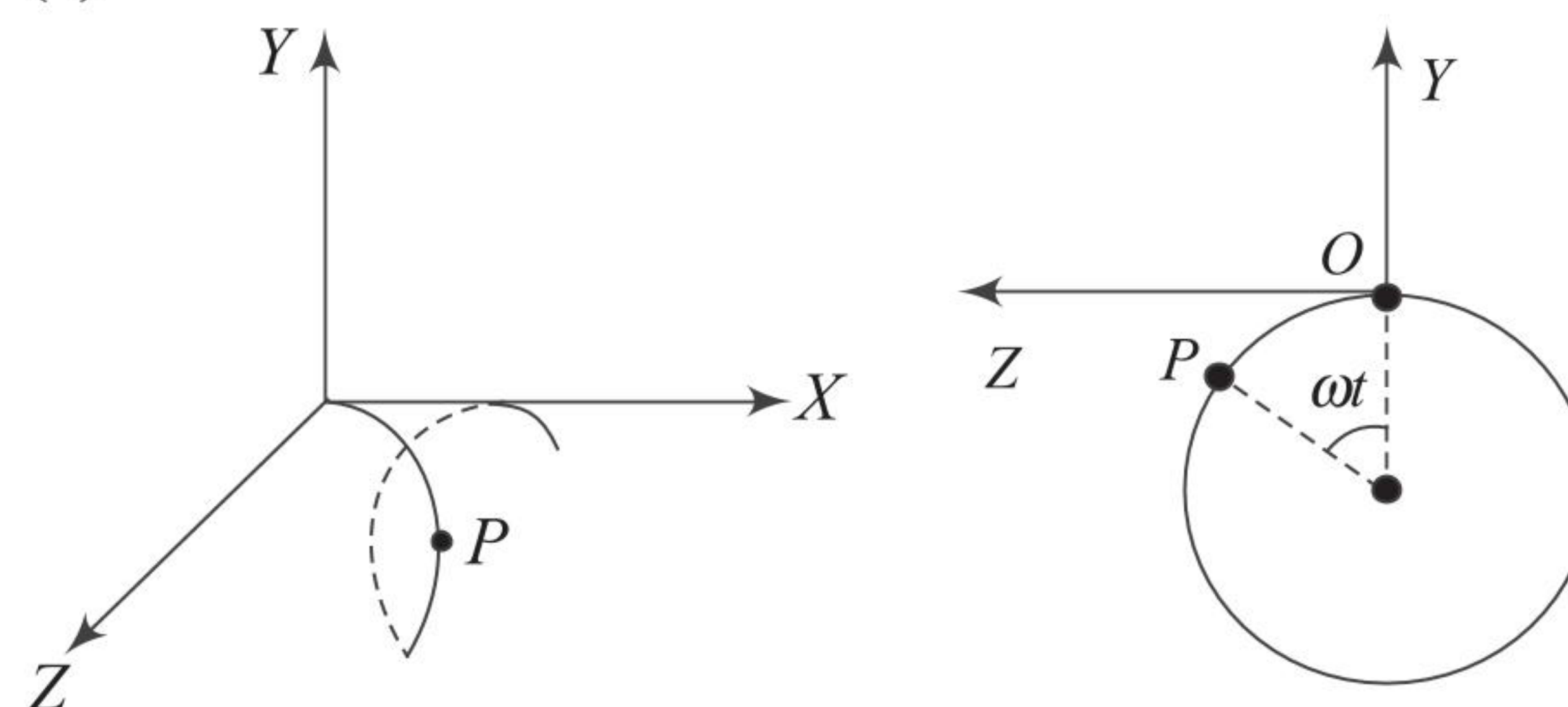
26. (1) At any time t the location of electron is shown as P . In two dimensional view of electron in YZ -plane, the situation is more clear.

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(h\nu - \phi)}{m}}$$

$$p = v \cos \theta = \frac{2\pi m}{qB_0}$$

$$pqB_0 = 2\pi \cos \theta m \sqrt{\frac{2(hf - \phi)}{m}} = 2\pi \cos \theta \sqrt{2m(hf - \phi)}$$

27. (2)



X-coordinate, $x = v \cos \theta \times t$

Y-coordinate, $y = -[R - R \cos \omega t]$

Z-coordinate, $z = R \sin \omega t$

So, $z = \frac{mv \sin \theta}{qB_0} \times \sin \left[\frac{qB_0}{m} t \right]$

$$= \frac{\sqrt{2m(hf - \phi)} \times \sin \theta}{qB_0} \times \sin \left[\frac{qB_0}{m} t \right]$$

28. (3) From $x = v \cos \theta \times t = \frac{\sqrt{2(hf - \phi)}}{m} \times \cos \theta \times t$

As f increases, slope of x versus t graph (a straight line) increases.

For Problems 29–31

29. (2) The energy of the incident photon is

$$E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{3300 \times 10^{-10}} = 3.75 \text{ eV}$$

A will not emit photoelectrons because energy of incident photon is less than work function of A .

30. (3) $eV_0 = h(f - f_0)$

When $V_0 = 0, f = f_0$, the threshold frequency.

From the graph it follows that $f_0 = 4 \times 10^{15}$ Hz

Therefore, work function is $\phi = hf_0 = 6.6 \times 10^{-34} \times 4 \times 10^{15}$
 $= 16.5$ eV

31. (1) $eV_0 = hf - hf_0$

$$V_0 = \frac{h}{e}f - \frac{h}{e}f_0$$

$$y = mx + c$$

$$\text{Slope} = \frac{h}{e}$$

For Problems 32–34

32. (3) $\frac{1}{2}mv^2 = eV$

$$v = \sqrt{\frac{2eV}{m}}$$

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2meV}}$$

$$\text{Fringe width } \beta = \frac{\lambda D}{d} = \frac{hD}{d\sqrt{2meV}}$$

33. (2) As v increases, λ decreases.

34. (3) Central maxima will remain at the same position.

For Problems 35–37

35. (2) Energy required, $E = ms \Delta\theta + mL$

$$= 2 \times 10^{-9} \times 4.2 \times 10^3 \times 70 + 2 \times 10^{-9} \times 2.25 \times 10^6$$

$$= 5.088 \times 10^{-3} \text{ J}$$

36. (2) Power output of the laser,

$$P = \frac{E}{t} = \frac{5.088 \times 10^{-3}}{450 \times 10^{-6}} = 11.3 \approx 11 \text{ W}$$

37. (1) $N = \frac{E}{hv} = \frac{E\lambda}{hc} = \frac{(3.18 \times 10^{16} \text{ eV})(585 \text{ nm})}{(1240 \text{ eV nm})} = 1.5 \times 10^{16}$

For Problems 38–39

38. (4) $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$ where $E = \text{kinetic energy}$

$$\frac{1}{\lambda_p} : \frac{1}{\lambda_d} : \frac{1}{\lambda_\alpha} = 1 : \frac{1}{\sqrt{2}} : \frac{1}{2} = 2 : \sqrt{2} : 1$$

39. (1) $\lambda = \sqrt{\frac{150}{V}} = \sqrt{\frac{150}{150}} \text{ \AA} = 1 \text{ \AA}$

Matrix Match Type

1. i. $\rightarrow$ a., c.; ii. $\rightarrow$ b., d.; iii. $\rightarrow$ d.; iv. $\rightarrow$ a., c.

(i) From $eV_0 = hf - \phi$ and $K_{\max} = hf - \phi$

If f increases keeping ϕ constant, then V_0 and $K_{\max}$ increase.

(ii) If I increases, more photons/time are incident on the metal surface and more photoelectrons would be liberated. Hence, saturation photocurrent increases. Stopping potential and $K_{\max}$ will remain the same.

(iii) If separation between cathode and anode is increased, then there is no effect on V_0 , $K_{\max}$ or current.

(iv) If ϕ decreases keeping f and I constant, then V_0 and $K_{\max}$ increase.

2. i. $\rightarrow$ d.; ii. $\rightarrow$ a., b., c., d.; iii. $\rightarrow$ b., c., d.; iv. $\rightarrow$ a., b., c., d.

(i) Maximum kinetic energy of ejected electron is given by Einstein's photoelectric equation $K_{\max} = hf - (hf/3) = 2hf/3$. As no potential difference is applied across target and collector and vacuum is there in the tube, so this maximum KE remains same at all locations.

(ii), (iii) and (iv) Kinetic energy of ejected photoelectrons can be anything from 0 to $K_{\max}$ (as found for (i)). It remains the same at all locations (reasoning is same as for above).

3. i. $\rightarrow$ c.; ii. $\rightarrow$ a., b.; iii. $\rightarrow$ a., b.; iv. $\rightarrow$ d.

(i) If intensity changes, then number of photons/time incident on the metal surface change and hence number of photoelectrons liberated change, so saturation photocurrent changes. Stopping potential and $K_{\max}$ will remain the same.

(ii) From $eV_0 = hf - \phi$ and $K_{\max} = hf - \phi$

If f changes, then V_0 and $K_{\max}$ change.

(iii) From $eV_0 = hf - \phi$ and $K_{\max} = hf - \phi$

If target material changes, then ϕ changes, then V_0 and $K_{\max}$ change.

(iv) If we change the potential difference between emitter and collector, then time taken for electrons to eject changes.

4. i. $\rightarrow$ a., c.; ii. $\rightarrow$ a., c.; iii. $\rightarrow$ b.; iv. $\rightarrow$ d.

Consider two equations.

$$eV_s = \frac{1}{2}mv_{\max}^2 = hf - \phi_0 \quad \dots(i)$$

Number of photoelectrons ejected per second $\propto$ intensity $\dots(ii)$

(i) As frequency is increased keeping intensity constant, V_s will increase and hence, $1/2m(v_{\max}^2)$ will increase.

(ii) As frequency is increased and intensity is decreased, V_s will increase and hence $1/2m(v_{\max}^2)$ will increase and saturation current will decrease.

(iii) If work function is increased, photoemission may stop.

(iv) If intensity is increased, then saturation current will increase.

5. i. $\rightarrow$ a., b., c., d.; ii. $\rightarrow$ a., b., c., d.; iii. $\rightarrow$ a, c.; iv. $\rightarrow$ a.

(i) Bichromatic light source is having two wavelengths and hence from energy point of view, two types of photons are possible. As light propagates in different directions, the photon can have different or same momenta depending upon the magnitude and direction of photon motion.

(ii) Same reasoning as for (i).

(iii) For a monochromatic light source, all photons have same energy but momenta can be different due to different directions.

(iv) Laser is a very narrow beam of monochromatic light, so all photons have nearly same energy and momenta.

Numerical Value Type

1. (1) $r \propto \frac{p}{q} \quad \left(\text{since, } r = \frac{p}{Bq} \right)$

where $p = \text{momentum}$, Given $r_\alpha = \frac{1}{2}r_e$

$$\frac{p_\alpha}{B2e} = \frac{1}{2} \left(\frac{p_e}{Be} \right)$$

or $p_\alpha = p_e$

$$\lambda \propto \frac{1}{p} \quad \left(\text{since, } \lambda = \frac{h}{p} \right)$$

$$\text{So, } \lambda_\alpha = \lambda_e$$

$$\text{or } n = 1$$

2. (4) Speed of photon (c) = $3 \times 10^8 \text{ ms}^{-1}$. Let λ be the wavelength of the photon. The de Broglie wavelength of the electron is h/mv .

$$\text{Given } \lambda = \frac{h}{mv}. \text{ Now,}$$

$$\frac{\text{KE of photon}}{\text{KE of electron}} = \frac{hf}{(1/2)mv^2} = \frac{2hc}{mv^2\lambda} = \frac{2c}{v} = \frac{2 \times 3 \times 10^8}{1.5 \times 10^8} = 4$$

$$\left(\therefore \lambda = \frac{h}{mv} \right)$$

3. (5) For K_α X-ray, $(Z-1)^2 \lambda = \text{constant}$.

$$\text{Hence, } (9-1)^2 \lambda = (Z-1)^2 (4\lambda)$$

$$(Z-1)^2 = \frac{64}{4} = 16$$

$$Z-1 = 4$$

$$\text{or } Z = 5$$

4. (4) The energy of each photon = $\frac{200 \text{ J/s}}{4 \times 10^{20} / \text{s}} = 5 \times 10^{-19} \text{ J}$

$$\text{Wavelength} = \lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34} \text{ J-s}) \times (3 \times 10^8 \text{ m/s})}{(5 \times 10^{-19} \text{ J})}$$

$$= 4.0 \times 10^{-7} \text{ m}$$

5. (1) $\lambda = 663 \times 10^{-9} \text{ m}$, $\theta = 60^\circ$, $n = 1.0 \times 10^{19}$

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{6.63 \times 10^{-9}} = 10^{-27}$$

Force exerted on the wall is

$$n \times 2 \times p \cos \theta = 2 \times 1 \times 10^{19} \times 10^{-27} \times \frac{1}{2} = 1 \times 10^{-8} \text{ N}$$

6. (1) Let n photons (each of frequency f) per second are emitted from source. Then power of source is $P = nhf$

But only 30% of the photons go towards mirrors.

Then force exerted on mirror is

$$F = 2 \left[\frac{30}{100} n \right] \frac{h}{\lambda} = \frac{3}{5} \frac{nhf}{c} = \frac{3}{5} \frac{P}{c}$$

and this force should be equal to weight of mirror,

$$\text{so } \frac{3}{5} \frac{P}{c} = 20 \times 10^{-3} \text{ g}$$

$$\Rightarrow P = \frac{5 \times 3 \times 10^8 \times 20 \times 10^{-3} \times 10}{3} = 10^8 \text{ W}$$

7. (3) Given $\lambda = 200 \text{ nm} = 2 \times 10^{-7} \text{ m}$

Energy of one photon is

$$\frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2 \times 10^{-7}} = 9.945 \times 10^{-19}$$

$$\text{Number of photons is } \frac{1 \times 10^{-7}}{9.945 \times 10^{-19}} = 1 \times 10^{11}$$

Hence, number of photoelectrons emitted is

$$\frac{1 \times 10^{11}}{10^4} = 1 \times 10^7$$

Net amount of +ve charge 'q' developed due to the outgoing electrons = $1 \times 10^7 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-12} \text{ C}$.

Now potential developed at the centre as well as at the surface due to these charges is

$$\frac{Kq}{r} = \frac{9 \times 10^9 \times 1.6 \times 10^{-12}}{4.8 \times 10^{-2}} = 3 \times 10^{-1} \text{ V} = 0.3 \text{ V}$$

8. (9) Given: Fringe width, $y = 1.0 \text{ mm} \times 2 = 2.0 \text{ mm}$

$$d = 0.24 \text{ mm}, W_0 = 2.2 \text{ eV}, D = 1.2 \text{ m}$$

$$y = \frac{\lambda D}{d}$$

$$\text{or } \lambda = \frac{yd}{D} = \frac{2 \times 10^{-3} \times 0.24 \times 10^{-3}}{1.2} = 4 \times 10^{-7} \text{ m}$$

$$E = \frac{hc}{\lambda} = \frac{4.14 \times 10^{-15} \times 3 \times 10^8}{4 \times 10^{-7}} = 3.105 \text{ eV}$$

$$\text{Stopping potential } eV_0 = 3.105 - 2.2 = 0.905 \text{ eV}$$

$$V_0 = \frac{0.905}{1.6 \times 10^{-19}} \times 1.6 \times 10^{-19} \text{ V} = 0.905 \text{ V}$$

9. (2) Momentum of electron, $p_e = \sqrt{2meV}$

$$\text{Momentum of particle, } p_p = \sqrt{2 \frac{m}{4} \cdot eV}$$

$$\therefore \frac{\lambda_p}{\lambda_e} = \frac{h/p_p}{h/p_e} = \sqrt{\frac{m}{m/4}} = 2$$

10. (38) Incident energy of photons = $\frac{hc}{200} = 6.2 \text{ eV}$

$$6.2 = K_1 + W_0, K_1 = 6.2 - W_0$$

$$K_2 = K_1 + 10 \text{ eV} = 16.2 - W_0$$

$$K_2 = \frac{hc}{\lambda_{\min}} = 12.4 \Rightarrow W_0 = 3.8 \text{ eV, given } W_0 = \frac{x}{10}$$

$$\therefore x = 38$$

11. (2.00)

$$K_{\max} = (5 - \phi) \text{ eV}$$

When electrons are accelerated through 5 V, they will reach the anode with energy = $(5 - \phi + 5) \text{ eV}$

$$\therefore 10 - \phi = 8$$

$$\text{or } \phi = 2 \text{ eV}$$

12. (0.50)

Momentum corresponding to incident photons normal to the surface,

$$\left(\frac{dp}{dt} \right)_{\text{incident}} = \frac{I}{c} dA \cos^2 \theta$$

Since reflection coefficient is ρ , so the momentum of the reflected photons per second normal to surface,

$$\left(\frac{dp}{dt} \right)_{\text{reflected}} = \frac{I}{c} dA \rho \cos^2 \theta$$

Hence, rate of change of momentum of the photons,

$$\left(\frac{dp}{dt} \right)_{\text{photons}} = -\frac{I}{c} dA (\rho - 1) \cos^2 \theta$$

$$\text{From Newton's third law, } F = \left(\frac{dp}{dt} \right)_{\text{surface}} = \frac{I}{c} dA (\rho + 1) \cos^2 \theta$$

$$\text{Hence, pressure exerted on surface, } P = \frac{dF}{dA} = \frac{I}{c} (\rho + 1) \cos^2 \theta$$

On substituting values, we get $P = 0.5 \text{ N cm}^{-2}$

13. (0.148)

Maximum kinetic energy of ejected electrons

$$K_{\max} = \frac{1}{2}mv_{\max}^2 = \frac{hc}{\lambda} - W$$

$$\therefore K_{\max} = \frac{(6.62 \times 10^{-34})(3 \times 10^8)}{(180 \times 10^{-9})} - (2 \times 1.6 \times 10^{-19})$$

$$= 7.8 \times 10^{-19} \text{ J}$$

$$\therefore v_{\max} = \sqrt{\left(\frac{2K_{\max}}{m}\right)} = \sqrt{\left(\frac{2 \times (7.8 \times 10^{-19})}{1.3 \times 10^{-31}}\right)} = 1.3 \times 10^6 \text{ m s}^{-1}$$

$$\text{Now, } Bev_{\max} = \frac{mv_{\max}^2}{r}$$

$$\Rightarrow r = \frac{mv_{\max}}{Be} = \frac{(9.1 \times 10^{-31})(1.3 \times 10^6)}{(5 \times 10^{-5})(1.6 \times 10^{-19})} = 0.148 \text{ m}$$

14. (0.63)

$$\text{de-Broglie wavelength, } \lambda = \frac{h}{p}$$

$$\text{Mean KE of helium atom} = \frac{3kT}{2}$$

$$\text{or } E = \frac{3 \times (1.38 \times 10^{-23}) \times 400}{2}$$

$$\text{In terms of momentum, the energy is given by } E = \frac{p^2}{2m}$$

$$\text{or } p^2 = 2mE$$

$$\text{or } p = \sqrt{2mE}$$

$$\therefore p = \sqrt{2 \times (4.002 \times 1.66 \times 10^{-27}) \times 3 \times (1.38 \times 10^{-23}) \times 200}$$

(where mass of helium $4.002 \text{ amu} = 4.002 \times 1.66 \times 10^{-27} \text{ kg}$)

$$\text{Now, } \lambda = \frac{6.625 \times 10^{-34}}{p}$$

Substituting the value of p and solving it, we get

$$\lambda = 0.63 \times 10^{-10} \text{ m}$$

15. (450)

We know that, de-Broglie wavelength

$$\lambda = \frac{h}{mv} \quad \text{and} \quad E = \frac{1}{2}mv^2$$

$$\therefore \lambda = \frac{h}{\sqrt{2mE}}$$

$$\text{In first case, } 100 \times 10^{-12} = \frac{h}{\sqrt{2mE_1}} \quad \dots (i)$$

$$\text{In second case, } 50 \times 10^{-12} = \frac{h}{\sqrt{2mE_2}} \quad \dots (ii)$$

$$\text{Dividing Eq. (i) by Eq. (ii), we get } 2 = \sqrt{\left(\frac{E_2}{E_1}\right)}$$

$$\text{or } E_2 = 4E_1$$

So, energy to be added $= 4E_1 - E_1 = 3E_1$

$$\text{Now, } \frac{h}{\sqrt{2mE_1}} = 100 \times 10^{-12}$$

$$\text{or } \sqrt{2mE_1} = \frac{6.625 \times 10^{-34}}{10^{-10}}$$

$$\text{or } \sqrt{2mE_1} = 6.625 \times 10^{-24}$$

$$\text{or } E_1 = \frac{(6.625 \times 10^{-24})^2}{2 \times (9.1 \times 10^{-31})} = 150 \text{ eV}$$

Therefore, energy added $= 3E_1 = 450 \text{ eV}$

16. (23.55)

Total number of ions in the rod

 $= \text{number of ions per unit volume} \times \text{volume of rod}$

$$= (2 \times 10^{25}) \times \{3.14 \times (0.005)^2 \times (5 \times 10^{-2})\}$$

$$= 7.85 \times 10^{19}$$

The number of photons excited in one direction is equal to the total number of ions because all ions are excited. Now, excited energy $= \text{number of excited photons} \times \text{energy of photon}$

$$= (7.85 \times 10^{19}) \times \frac{hc}{\lambda}$$

$$= (7.85 \times 10^{19}) \times \frac{(6.6 \times 10^{-34}) \times (3 \times 10^8)}{6.6 \times 10^{-7}}$$

$$= 23.55 \text{ W}$$

Archives

JEE Advanced

Single Correct Answer Type

$$1. (2) \quad t = 100 \times 10^{-9} \text{ s}, P = 30 \times 10^{-3} \text{ W}, c = 3 \times 10^8 \text{ m s}^{-1}$$

$$\text{Momentum} = \frac{Pt}{c} = \frac{30 \times 10^{-3} \times 1000 \times 10^{-9}}{3 \times 10^8}$$

$$= 1.0 \times 10^{-17} \text{ kg m s}^{-1}$$

$$2. (1) \quad \frac{K_1}{K_2} = \left(\frac{u_1}{u_2}\right)^2 = 4$$

$$K_1 = \frac{hc}{\lambda} - \phi = \frac{1240}{248} - \phi$$

$$K_1 = (5 - \phi)$$

$$K_2 = \frac{1240}{310} - \phi$$

$$K_2 = (4 - \phi)$$

$$\frac{K_1}{K_2} = \left(\frac{5 - \phi}{4 - \phi}\right) = 4 \Rightarrow 5 - \phi = 4(4 - \phi)$$

$$\Rightarrow 5 - \phi = 16 - 4\phi$$

$$\Rightarrow 3\phi = 11 \Rightarrow \phi = \frac{11}{3} = 3.7 \text{ eV}$$

$$3. (2) \quad K_{\max} = \frac{hc}{\lambda} - \phi$$

$$eV_s = \frac{hc}{\lambda} - \phi$$

$$1.6 \times 10^{-19} \times 2 = \frac{h \times 3 \times 10^8}{3000 \times 10^{-10}} - \phi \quad \dots (i)$$

$$1.6 \times 10^{-19} \times 1 = \frac{h \times 3 \times 10^8}{4000 \times 10^{-10}} - \phi \quad \dots (ii)$$

$$\text{From Eq. (ii), } \phi = \frac{h \times 3 \times 10^8}{4000 \times 10^{-10}} - 1.6 \times 10^{-19}$$

$$1.6 \times 10^{-19} \times 2 = \frac{h \times 3 \times 10^8}{3000 \times 10^{-10}} - \frac{h \times 3 \times 10^8}{4000 \times 10^{-10}} + 1.6 \times 10^{-19}$$

$$1.6 \times 10^{-19} = \frac{h \times 3 \times 10^8}{10^{-7}} \left(\frac{1}{3} - \frac{1}{4}\right) = \frac{h \times 3 \times 10^8}{10^{-7}} \left[\frac{4-3}{12}\right]$$

$$1.6 \times 10^{-19} = \frac{h \times 3 \times 10^8}{10^{-7}} \times \frac{1}{12}$$

$$1.6 \times 4 \times \frac{10^{-19} \times 10^{-7}}{10^8} = h$$

$$6.4 \times 10^{-34} \text{ Js} = h$$

4. (2) λ_e = Minimum de Broglie wave length

$$\frac{hc}{\lambda_{ph}} = \phi + \frac{hc}{\lambda_e} + eV$$

$$hc \left(\frac{-1}{\lambda_{ph}^2} d\lambda_{ph} \right) = hc \left(-\frac{1}{\lambda_e^2} d\lambda_e \right)$$

$$\frac{d\lambda_{ph}}{d\lambda_e} = \frac{\lambda_{ph}^2}{\lambda_e^2}$$

$$V \gg \frac{\phi}{e}$$

Energy of electron = eV

$$\frac{p^2}{2m} = E \quad \lambda = \frac{h}{p} = \frac{1}{\sqrt{2mE}}$$

$$p = \sqrt{2mE}$$

5. (1) According to photoelectric effect equation:

$$K_{\max} = \frac{hc}{\lambda} - \phi_0$$

$$\Rightarrow \frac{p^2}{2m} = \frac{hc}{\lambda} - \phi_0$$

$$\Rightarrow \frac{(h/\lambda_d)^2}{2m} = \frac{hc}{\lambda} - \phi_0$$

Assuming small changes, differentiating both sides,

$$\frac{h^2}{2m} \left(-\frac{2}{\lambda_d^3} \right) d\lambda_d = -\frac{hc}{\lambda^2} d\lambda \Rightarrow \frac{d\lambda_d}{d\lambda} \propto \frac{\lambda_d^3}{\lambda^2}$$

Linked Comprehension Type

$$1. (1) a = \frac{n\lambda}{2} \Rightarrow \lambda = \frac{2a}{n}$$

$$\lambda_{\text{de Broglie}} = \frac{h}{p}$$

$$\frac{2a}{n} = \frac{h}{p} \Rightarrow p = \frac{nh}{2a}$$

$$E = \frac{p^2}{2m} = \frac{n^2 h^2}{8a^2 m}$$

$$\Rightarrow E \propto 1/a^2$$

$$2. (2) E = \frac{h^2}{8a^2 m} = \frac{(6.6 \times 10^{-34})^2}{8 \times (6.6 \times 10^{-9})^2 \times 10^{-30} \times 1.6 \times 10^{-19}} = 8 \text{ meV}$$

$$3. (4) mv = \frac{nh}{2a}$$

$$\text{or } v = \frac{nh}{2am}$$

$$\Rightarrow v \propto n$$

Numerical Value Type

$$1. (3) \text{ de-Broglie wavelength, } \lambda = \frac{h}{\sqrt{2mK}}$$

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

[$\therefore$ Kinetic energy, $K = qV$]

$$\lambda_p = \frac{h}{\sqrt{2m_p q_p V}}, \lambda_\alpha = \frac{h}{\sqrt{2m_\alpha q_\alpha V}}$$

$$\begin{aligned} \therefore \frac{\lambda_p}{\lambda_\alpha} &= \frac{h}{\sqrt{2m_p q_p V}} \times \frac{\sqrt{2m_\alpha q_\alpha V}}{h} \\ &= \frac{\sqrt{m_\alpha q_\alpha}}{\sqrt{m_p q_p}} = \frac{\sqrt{(4m_p)(2q_p)}}{\sqrt{(m_p q_p)}} = \sqrt{8} = 2\sqrt{2} \approx 3 \end{aligned}$$

2. (7) $R = 1 \text{ cm}$

$$f = 4.7 \text{ cm}$$

$$\frac{hc}{\lambda} = \phi + eV$$

$$\frac{1240(\text{eV})(\text{nm})}{200(\text{nm})} = 4.7(\text{eV}) + eV$$

$$\frac{1240}{200} e = 4.7e + eV$$

$$6.2 - 4.7 = V$$

$$\therefore V = 1.5 \text{ volt}$$

$$\frac{1}{4\pi\epsilon_0} \frac{Q}{R} = 1.5$$

$$(9 \times 10^9) \frac{Ne}{1} = 1.5$$

$$9 \times 10^{11} Ne = 1.5$$

$$N = \frac{1.5}{9 \times 10^{11} \times 1.6 \times 10^{-19}} = \frac{15}{16} \times \frac{1}{9} \times 10^8$$

$$= \frac{5}{3 \times 16} \times 10^8 = \frac{50}{48} \times 10^7$$

$$\therefore Z = 7$$

3. (7)

$$+120e \quad r = 10 \text{ fm} \quad +e$$

$$\frac{(9 \times 10^9)(120e)(e)}{10 \times 10^{-15}} = \frac{p^2}{2m}$$

$$\lambda = \frac{h}{p}$$

$$\begin{aligned} \lambda^2 &= \frac{h^2}{p^2} = \frac{h^2 \times 10 \times 10^{-15}}{2m \times 9 \times 10^{-9} \times 120e^2} \\ &= \frac{4.2 \times 4.2 \times 10^{-30} \times 10 \times 10^{-15}}{2(5/3) \times 10^{-27} \times 9 \times 10^{-9} \times 120} \\ &= \frac{42 \times 42}{36} \times 10^{-30} \end{aligned}$$

$$\Rightarrow \lambda = 7 \times 10^{-15} \text{ m} = 7 \text{ fm}$$

4. (1) According to Einstein's photoelectric equation, $K_{\max} = h\nu - \phi_0$ where the symbols have their usual meaning.

But $K_{\max} = eV_s$ where V_s is the stopping potential $V_s = \frac{h}{e}\nu - \frac{\phi_0}{e}$

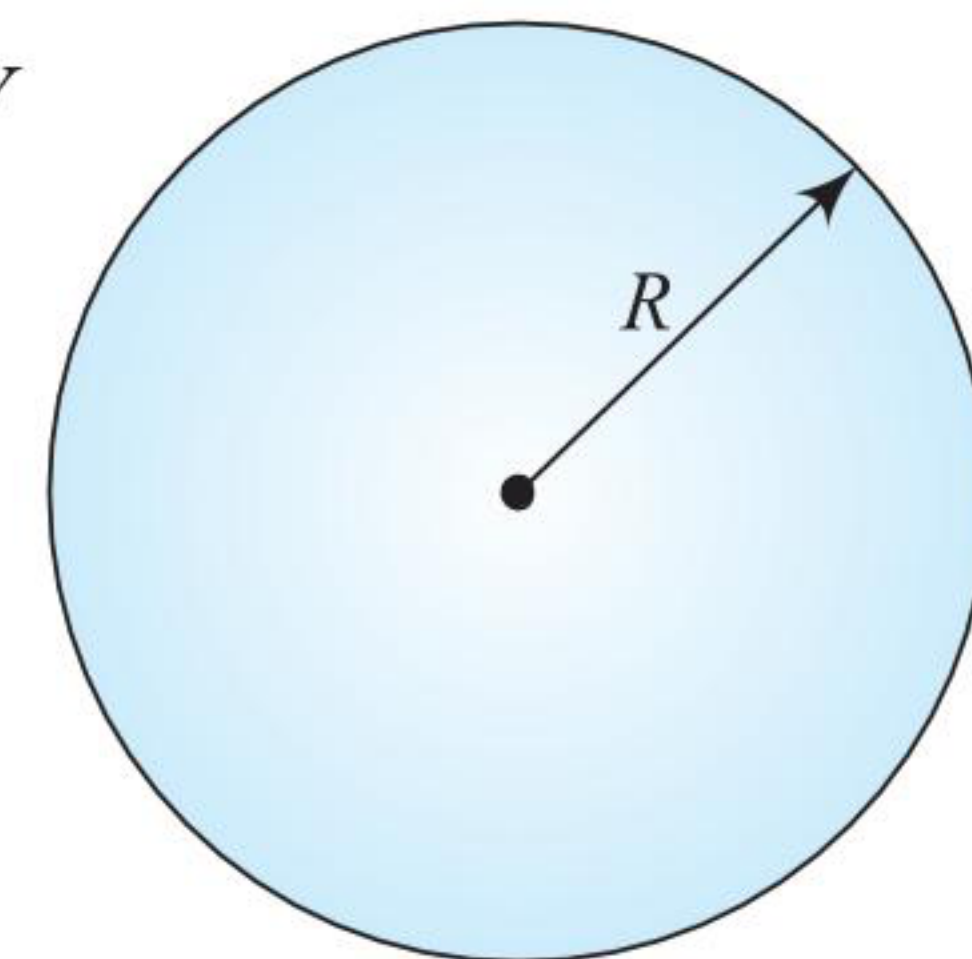
Thus, the graph between $V_s - \nu$ is a straight line.

Compare the above relation with $y = mx + C$

$$\therefore \text{Slope of } V_s - \nu \text{ graph} = \frac{h}{e}$$

It is same for both the metals.

$$\therefore \text{Ratio of the slopes} = 1$$



$$5. (24) \text{ Power} = nhf, n = \text{number of photons per second} = \frac{200 \text{ W}}{6.25 \text{ eV}}$$

$$= \frac{200}{1.6 \times 10^{-19} \times 6.25} = \frac{200}{6.25 \times 1.6 \times 10^{-19}}$$

As photon is just above threshold frequency, KE_{max} of the electron released is zero and they are accelerated by potential difference of 500 V.

$$\frac{p^2}{2m} = q\Delta V \Rightarrow p = \sqrt{2mq\Delta V}$$

Since efficiency is 100%, number of electrons = number of photons per second. As photon is completely absorbed force exerted $F = nmv = np$.

$$\text{Hence, } F = \frac{200}{6.25 \times 1.6 \times 10^{-19}} \times \sqrt{2(9 \times 10^{-31}) \times 1.6 \times 10^{-19} \times 500}$$

$$= \frac{3 \times 200 \times 10^{-25} \sqrt{1600}}{6.25 \times 1.6 \times 10^{-19}}$$

$$= \frac{2 \times 40}{6.25 \times 1.6} \times 10^{-4} \times 3 = 24.00$$

6. (1.00)

If v is speed of mirror just after absorption of photons

$$Mv - N \frac{2h}{\lambda} = 0 \Rightarrow v = \frac{2Nh}{M\lambda}$$

If L is the maximum compression in the spring

$$\frac{1}{2}kL^2 = \frac{1}{2}mv^2$$

$$\Rightarrow v = \sqrt{\frac{k}{m}} L$$

$$\Rightarrow \frac{2Nh}{M\lambda} = \Omega \times 10^{-6}$$

$$\Rightarrow N = \Omega \times 10^{-6} \times \frac{M\lambda}{2h} = 8\pi \times 10^{-6} \times 10^{-6} \times \frac{M\lambda}{2h}$$

$$\frac{4\pi M\lambda}{h} \times 10^{-12} = 10^{24} \times 10^{-12} = 10^{12}$$

$$\Rightarrow x = 1$$

7. (6) Energy of incident photons should be equal to summations of work function and maximum kinetic energy of the photons.

$$E = \phi + K_{\text{max}}$$

$$E_0 = \phi_P + E_P = 4 + E_P \quad \dots(i)$$

$$E_0 = \phi_Q + E_Q = 4.5 + E_Q \quad \dots(ii)$$

$$E = \phi_R + E_R \quad \dots(iii)$$

As same source of light is used for P and Q

From (i) and (ii)

$$4 + E_P = 4.5 + E_Q$$

$$E_P = 2E_R = 2E_Q$$

$$4 + E_P = 4.5 + E_Q$$

$$4 + 2E_P = 4.5 + E_Q$$

$$E_Q = 0.5 \text{ eV}$$

$$E_P = 2 \times 0.5 = 1 \text{ eV}$$

$$E_R = \frac{E_P}{2} = \frac{1}{2} = 0.5$$

$$E = \phi_R + E_R = 5.5 + 4.5 = 6.0 \text{ eV}$$